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The diagnosis of complex rotation in the lightcurve of 4179 Toutatis and potential applications to other asteroids and bare cometary nuclei

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Abstract

The analysis of radar observations of the asteroid 4179 Toutatis by Hudson and Ostro (1995) yielded a complex spin state. We revisit the visible lightcurve data on Toutatis (Spencer et al., 1995) to explore the feasibility of using a rotational lightcurve to recover the signature of an excited spin state. For this, we apply Fourier transform and CLEAN algorithms (WindowCLEAN). WindowCLEAN yields clear and precise frequency signatures associated with the precession of the long axis about the total angular momentum vector and a combination of this precession and rotation about the long axis. For a long axis mode (LAM) state, our periodicities for Toutatis yield a mean long axis precession period, $P_\phi$, of 7.38 dy and a rotation period around the long axis, $P_\psi$, of 5.38 dy which compare well with the respective periods of 7.42 dy and 5.37 dy derived by Ostro et al. (1999) and represent an independent confirmation of these values. We explain why the dramatic change in the Earth-Toutatis-sun geometry during the time that the lightcurve was obtained, has little effect on the final results obtained. Using the Toutatis example as a guide, we discuss the capabilities as well as the limitations on deriving information about complex spin states from asteroidal lightcurves.

Key words: Asteroids, Rotational Dynamics; 4179 Toutatis
1 Introduction

4179 Toutatis is one of the two small bodies in the solar system with a confirmed non-principal axis rotation (the other being 1P/Halley), and the first asteroid discovered to be in such a state. Its very close approach to Earth in December 1992 (0.024 AU), enabled an extensive time series of radar images to be acquired and ultimately the determination of its shape and spin state (Hudson and Ostro 1995). The elongated asteroid was found to be spinning in a long-axis mode (LAM) state consisting of a precession of the long axis about the rotational angular momentum vector with a mean period \( P_\phi \) [frequency] of 7.35 dy \([0.136 \text{dy}^{-1}]\) and a rotation of the long-axis around itself with a period \( P_\psi \) [frequency] of 5.41 dy \([0.185 \text{dy}^{-1}]\). Hudson and Ostro (1998) confirmed their radar based shape and dynamical model by showing that, with a suitable selection of light scattering parameters, they can account for the complex visible lightcurve observed by Spencer et al. (1995). In their latest work, Ostro et al. (1999) refined the periods of \( P_\phi \) and \( P_\psi \) to 7.42 dy and 5.37 dy, respectively.

Work by Kryszczyńska et al. (1999) based on their model lightcurve is intended to show that the signature of complex rotation and the periodicities found by Spencer et al. (1995) are consistent with the above mentioned results of Hudson and Ostro (1998). Spencer et al. (1995) found periodicities [frequencies] in their December 92 - January 93 data near 7.4 dy \([0.135 \text{dy}^{-1}]\) and 3.1 dy \([0.323 \text{dy}^{-1}]\) using the phase dispersion minimization (PDM) method. However, their confidence in these results was apparently not high enough to say more than: “The likely explanation is that Toutatis has complex, tumbling, rotation with a characteristic period between 3 dy and 7 dy.” In particular, the weakly determined 3.1 dy periodicity is apparently in conflict with the Hudson and Ostro (1995) results. Kryszczyńska et al. (1999) show by Fourier analysis of a model lightcurve that the 3.1 dy periodicity seen by Spencer et al. (1995) is a natural outcome which manifests itself in the Fourier spectrum at half the 3.1 dy periodicity. In fact, this is a reflection of the superposition of precession and rotation of the long axis as seen by an inertial observer. Thus from Spencer et al.’s (1995) 7.5 dy and 3.1 dy periodicities, a rotation period around the long axis of 5.3 dy can be deduced, thereby explaining the apparent conflict. Note that Kryszczyńska et al.’s (1999) work where the Euler angles \( \theta, \phi, \) and \( \psi \) are defined as in Landau and Lifshitz (1976) contains at least one major error: the period of the angle \( \theta \) is not precisely equal to that of \( \psi \) (see their figure 2), which rigid body dynamics requires for a LAM (this can be proved by a derivation analogous to that in the Appendix of Samarasinha and A’Hearn 1991 with the relevant Euler angles). Despite that, Kryszczyńska et al.’s (1999) work on Toutatis inspired us to analyze the original lightcurve data (Spencer et al. 1995).

During the time interval covered by the lightcurve, the observing and illumination conditions changed dramatically (see Spencer et al. 1995 for a detailed description). Earlier we were of the opinion that spurious frequencies could be introduced into the Fourier
transform by these effects and would undermine a Fourier based analysis. In section 2 we perform a WindowCLEAN analysis on the Spencer et al. (1995) data and show how the periodicities latent in Toutatis’ lightcurve can be obtained with considerable precision. In section 3 we give an explanation of why the rapid changes in viewing geometry have minimal impact on the analysis and, finally, in section 4 we outline how what has been learned from Toutatis can be used as a guide to diagnose complex rotation in lightcurves of asteroids and bare cometary nuclei.

2 WindowCLEAN analysis of the 4179 Toutatis data

The WindowCLEAN analysis has previously been used to derive periodicities and rotational models for comets 1P/Halley (Belton et al. 1991) and 29P/Schwassmann-Wachmann 1 (Meech et al. 1993). It has also been used to detect a change in the period of the nucleus of 10P/Tempel 2 (Mueller and Ferrin 1996).

The following description outlines the WindowCLEAN analysis which incorporates both a discrete Fourier transform and the CLEAN algorithm (Roberts et al. 1987). Additionally, in WindowCLEAN (first used by Belton and Gandhi, 1988), we introduce a cross correlation between the spectral window and the residual spectrum in the cleaning process that helps stabilize the algorithm in the case of noisy data. A discrete Fourier transform of the lightcurve data yields a dirty spectrum of the periodicities latent in the observations. The spectrum is called dirty because each intrinsic periodicity introduces an associated spectrum of aliases and all of these periodicities are present, together with unpredictable spurious periodicities that may occur due, perhaps, to the distribution of the data points in the time interval considered. A spectral window corresponding to the spectrum of aliases induced by the gaps in the observations is computed. The CLEAN algorithm uses the information in this spectral window to numerically (and approximately) remove the aliases from the dirty spectrum to form a clean spectrum that contains only the true periodicities and occasionally a few, mostly low power, spurious periodicities. These spurious periodicities can often be identified by applying WindowCLEAN to subsets of the data (or different data sets) and comparing the results with each other (see Foster 1995 for a review of these techniques).

2.1 December, 1992 – January, 1993 data

Absolute magnitudes H(1,1,0) (i.e. observed V magnitude corrected for heliocentric and geocentric distances of 1 AU and zero solar phase angle) were taken from Spencer et al. (1995). A Fourier transform (FT) was applied to the data yielding the dirty power spectrum shown in Figure 1a along with the spectrum of the data window. Most of the frequencies (but not the strongest) that are present in the dirty spectrum are aliases. The application
of the WindowCLEAN algorithm to the dirty spectrum yields the clean spectrum as shown in Figure 1b. We also show the spectrum of residuals in the cleaning process to confirm that all of the significant frequencies are accounted for. In the clean spectrum the number of significant peaks well above the noise has been reduced to four with two additional minor peaks. In falling order of power these occur at frequencies of $f_1 = 0.271\, \text{dy}^{-1}$, $f_2 = 0.135\, \text{dy}^{-1}$, $f_3 = 0.643\, \text{dy}^{-1}$, $f_4 = 0.029\, \text{dy}^{-1}$, $f_5 = 0.481\, \text{dy}^{-1}$, and $f_6 = 0.683\, \text{dy}^{-1}$.

The errors for the frequencies are not easily determined but an upper limit for the error can be estimated by using the standard deviation, $\sigma$, of the Gaussian used to describe the peaks in the power spectrum. This gives a measure of the probability that each peak can be found at respective locations which is mostly dependent on the total time baseline and only slightly on the quality or quantity of the data. In the clean spectrum of Toutatis, $\sigma$ is $0.007\, \text{dy}^{-1}$ for all peaks. The resultant upper limits of the errors in the component periods vary from $0.14\, \text{dy}$ to $0.19\, \text{dy}$.

### 2.2 WindowCLEAN analysis of model lightcurves

We will digress to explain the power spectrum of WindowCLEAN for non-principal axis rotators which is an extension of work by Mueller (1994). We used our MODELSIM program to generate model lightcurves for a triaxial shape (near prolate), different spin states and different orientations (mimicked by different Right Ascensions and Declinations for the angular momentum vector, which have been arbitrarily chosen). MODELSIM uses shape models, Hapke scattering parameters, and the spin state to calculate lightcurves. MODELSIM lightcurves and images have been used by us for 1P/Halley (Belton et al. 1991) and 10P/Tempel 2 (Mueller and Ferrin 1996). We used an elongated triaxial ellipsoid with semi-axes 2.80 km, 1.55 km, 1.25 km for the shape for all model lightcurves. In this work, the Euler angles $\theta$, $\phi$, and $\psi$ have the following significance: $\theta$ and $\phi$ give the orientation of the long axis relative to an inertial frame and $\psi$ measures the angular distance around the long axis. $\theta$ is measured from the direction of the rotational angular momentum. $P_\phi$ represents the mean period of precession of the long axis around the angular momentum vector while $P_\psi$ denotes the period of rotation or oscillation (for LAM and SAM, respectively) of the long axis around itself (see Belton 1991, Samarasinha and A’Hearn 1991). Table I lists the different scenarios considered. The Right Ascension (37 deg) is the same for all models.

The continuous lightcurves were calculated for three consecutive days. Additionally, for the LAM-1a case, we introduced gaps in the lightcurve and deleted data points to simulate an object that is only observable at night and take into account that uneven observational intervals are present. Only one model was considered for a SAM, because $P_\phi < P_\psi$ (Samarasinha and A’Hearn 1991).

Figure 2 shows the results of the WindowCLEAN for the LAM-1 case. The top is the result for LAM-1a, the middle for LAM-1a with gaps in the lightcurve, and the bottom
Figure 1: a. Dirty (top) and window spectrum (bottom) of the December/January data of 4179 Toutatis.
Figure 1: b. Clean (top) and residual spectrum (bottom) of the December/January data of 4179 Toutatis.
for LAM-1b. Three common peaks are present in all three spectra. The dominant peak for LAM-1a is at frequency $1/P_\phi$, the second peak is at $(2/P_\phi + 2/P_\psi)$ and a small peak is at $2/P_\phi$. Using model LAM-1a with observational gaps we see those same peaks and two additional ones at $(1/P_\phi + 1)$ and at $(2/P_\phi + 2/P_\psi + 1)$ which are daily aliases that were not cleaned out entirely. For LAM-1b the dominant peak is at $(2/P_\phi + 2/P_\psi)$, followed by $2/P_\phi$ and $1/P_\phi$. In all three cases, peaks at $1/P_\phi$, $2/P_\phi$, and $(2/P_\phi + 2/P_\psi)$ are present. We can therefore say (at least for these examples) that if we have three or more peaks, two of which have a 2:1 frequency ratio, that those two are most likely $2/P_\phi$ and $1/P_\phi$ with the third peak at $(2/P_\phi + 2/P_\psi)$. The other peaks are explained with daily aliases. For all LAM-1 cases the Earth (i.e. the observer) is outside the cone that the precessional motion of the long axis sweeps out.

Figure 3 shows the cleaned spectra for LAM-2a (top) and LAM-2b (bottom). Five peaks are present for LAM-2a at frequencies (with falling power) $1/P_\phi$, $2/P_\phi$, $(2/P_\phi + 2/P_\psi)$, $3/P_\phi$, and $(1/P_\phi + 2/P_\psi)$. For LAM-2b we get three peaks at $2/P_\phi$, $(2/P_\phi + 2/P_\psi)$, and $1/P_\phi$. The same conclusions can be reached as for the LAM-1 cases. For LAM-2a, the Earth is just at the edge of the cone swept out by the precessional motion and for LAM-2b it is clearly outside. This may explain the different relative strengths of the frequency peaks in the two cases.

Figure 4 shows the results of the WindowCLEAN for SAM-a (top) and SAM-b (bottom). The major peak for SAM-a is at $2/P_\phi$ and the second peak is at $(1/P_\phi + 1/P_\psi)$. We have only one peak at $2/P_\phi$ for SAM-b. Obviously the orientation in SAM-b is such that the lightcurve does not show the variations of $P_\psi$ and therefore the non-principal axis components cannot be recovered. The Earth is away from the direction of the angular momentum vector in both SAMs.

What do the above examples tell us? WindowCLEAN is capable of recovering the
Figure 2: Clean spectrum for model lightcurves LAM-1a (top), LAM-1a with gaps (middle), and LAM-1b (bottom).
Figure 3: Clean spectrum for model lightcurves LAM-2a (top) and LAM-2b (bottom).
Figure 4: Clean spectrum for model lightcurves SAM-a (top) and SAM-b (bottom).
correct period components of non-principal axis rotation for the cases we considered. If several peaks are present and two are 1:2 multiples of each other, one is \(1/P_\phi\) and the other is \(2/P_\phi\). If \(1/P_\phi\) is absent, \(2/P_\phi\) is likely to be the dominant frequency. An additional peak is most likely \((2/P_\phi + 2/P_\psi)\) if its frequency is greater than \(2/P_\phi\) or \((1/P_\phi + 1/P_\psi)\) if its frequency is less than \(2/P_\phi\).

These illustrations demonstrate the complexity involved in the proper interpretation of periodicity signatures in lightcurves. We will present a more in depth exploration of the parameter space in another paper. However, we can state confidently that the periodicities found with WindowCLEAN are always related to the physical periods of the object and not spurious unrelated periods. Kaasalainen (2001) investigated a limited sample of LAM and SAM model lightcurves and he also states that prominent peaks in the corresponding power spectra are usually found at \(2/P_\phi\) and \((2/P_\phi + 2/P_\psi)\) for LAMs, and \(2/P_\phi\) and \((2/P_\phi - 2/P_\psi)\) for SAMs. Note that he uses the same Euler angles as we do for the LAM cases but a different set for the SAM cases.

We did not include noise in our simulations. Adding noise will not change the outcome for our SAM model but might for LAM-1 as follows. In LAM-1a, if \(2/P_\phi\) gets lost in the noise than we would misidentify the major peak with \(2/P_\phi\) instead of \(1/P_\phi\) and thus determine \(P_\psi\) wrong as well. In LAM-1b, if the two peaks at \(1/P_\phi\) and \(2/P_\phi\) are not recoverable due to noise, we would not suspect a non-principal axis rotator. The same problem arises for LAM-2a, if \((2/P_\phi + 2/P_\psi)\) was lost in the noise. The lesson from this is that one needs high quality data at different observing geometries or over a very long time period to accurately recover the periods, both which were true for Toutatis.

2.3 Interpretation of the December, 1992 – January, 1993 data

For the Toutatis spectrum (Figure 1b), \(f_1 = 2f_2\), so we identify \(f_1\) with \(2/P_\phi\). Thus Toutatis’ long axis is precessing around the angular momentum vector with a frequency [period] of \(f_1/2 = 0.1355\) d\(^{-1}\) [7.38 d]. Given the above discussion, \(f_3\) is most likely \((2/P_\phi + 2/P_\psi)\). Taking \(P_\phi = 7.38\) d we deduce that \(P_\psi = 5.38\) d. Thus Toutatis rotates about the long axis once in 5.38 d. A SAM state is not possible in the case of Toutatis because \(P_\phi > P_\psi\) which is not dynamically permitted for SAM states (Samarasinha and A’Hearn 1991). \(f_4\) is equivalent to a periodicity of 34 d, and, by inspection, we identify this with the curvature of the envelope of the maxima of the lightcurve. \(f_5\) may be related to the combination \((2f_1 - 2f_4)\) or \((2/P_\phi + 1/P_\psi)\) and \(f_6\) may be \((f_3 + f_4)\). Clearly, \(f_5\) and \(f_6\) are related to the other frequencies. The results for \(P_\phi\) and \(P_\psi\) are in excellent agreement with the recently refined radar result by Ostro et al. (1999) who find \(P_\phi = 7.42\) d and \(P_\psi = 5.37\) d. Our work is the first to show that these periodicities can be recovered accurately directly from the visible lightcurve data for Toutatis.
2.4 Synthetic lightcurve corresponding to the December, 1992 – January, 1993 data

The use of our MODELSIM code here is not to compare model lightcurves with each other but to use model lightcurves to show how confidently we can use our WindowCLEAN analysis to recover periodicities of complex rotation in observed lightcurves. We used the initial spin conditions given in Hudson and Ostro (1995) and Hapke parameters from Hudson and Ostro (1998) to create a model lightcurve. The shape, however, was a homogeneous tri-axial ellipsoid with the semi-axes (i.e. dynamical principal axes) of 1.70 km, 2.03 km and 4.26 km given by Hudson and Ostro (1995). The resulting lightcurve shown in Figure 5 follows the trend of Spencer et al. (1995) observations very well except for the first few data points which were obtained at high phase angles (90-120 deg) where the errors due to an ellipsoid approximation to Toutatis’s irregular shape could be significant. We want to emphasize that our ellipsoid model lightcurve was not adjusted in time, absolute magnitude or amplitude. A comparison of our model lightcurve with the full radar shape model lightcurve of Hudson and Ostro (1998) shows very good agreement between the two (Fig. 6). Applying WindowCLEAN to our model lightcurve gives the clean spectrum shown in Figure 7. The strongest peaks fall at 0.135 dy$^{-1}$, 0.270 dy$^{-1}$, 0.642 dy$^{-1}$, and 0.024 dy$^{-1}$, again in excellent agreement with the frequencies found in the actual lightcurve data.

2.5 July – August, 1992 data

The same approach as for the December/January data was taken, but the data from Spencer et al. (1995) in July and August 1992 are much sparser. The clean spectrum is shown in Figure 8 and shows the strongest response at 0.246 dy$^{-1}$. Other peaks are present at much lower power and are not Gaussian. These peaks appear to be spurious. Using the data from July 23 - August 4, therefore avoiding a big gap in the observations between the beginning and the end of August, gives the same major peak but different minor peaks (not shown). The data are obviously too sparse to recover a signature of complex rotation. Nevertheless, it is encouraging that the frequency of the strongest peak is reasonably close to that recovered from the December/January data.

3 Discussion

It seemed remarkable to us that the rapid change in the aspect angle (nearly 90 deg) of Toutatis during the December/January observations would not affect the outcome of the WindowCLEAN analysis. The asteroid did not move appreciably in ecliptic latitude during the closest approach in December 1992 and January 1993 and therefore the change in longitude is a measure of the changing aspect angle (Spencer et al. 1995). Grouping
Figure 5: Our model lightcurve for 4179 Toutatis (solid line) and data (solid circles).
Figure 6: Comparison of our model lightcurve (solid line) with the model lightcurve from Hudson and Ostro (1998) (dashed line). Note that the magnitude has not been corrected for phase angle effects.
Figure 7: Clean spectrum of our model lightcurve for 4179 Toutatis.
Figure 8: Clean spectrum of the July/August data of 4179 Toutatis. Note, that except for the strongest peak, other signatures are not Gaussian.
the data together into subsets where the change in ecliptic longitude is less than 10 deg does not change the periods recovered in the individual subsets (each subset had a time baseline larger than the component periods). However, this approach was not possible in early December where the ecliptic longitude changed by more than 10 deg per day! We also grouped the data into two parts, from December 8–17, 1992, where the ecliptic longitude changed 65 deg, and from December 20, 1992 until January 28, 1993, with a change in longitude of 15 deg. This period search did not lead to a change in recovered periodicities for the second interval whereas for the first interval, the three frequencies recovered were 0.301 dy⁻¹, 0.095 dy⁻¹, and 0.447 dy⁻¹. These do not seem related to the frequencies recovered from the other time intervals, i.e., the rapid change in aspect angle prevented the recovery of the correct periodicities.

Figure 9 shows the evolution of the sub-solar and the sub-Earth directions during the December/January observations. While the rate of change of the sub-solar direction varied only slightly during the whole observing interval, the change in the sub-Earth direction was rapid in early December (i.e., during the time closest to perigee). As the rotational motion is retrograde with respect to the orbital motion (Hudson and Ostro 1995), the synodic periods should be smaller than the sidereal periods. We estimate that their differences should be smaller than 3%. However, complicating the issue is the fact that the changes in sub-solar and sub-Earth directions are approximately opposite to each other, therefore partially nullifying the synodic effects on the periods. Nevertheless, we can safely assume that a strict upper limit to the difference between the lightcurve derived periods and the real periods is 3%. The error in the frequency peaks derived from the lightcurve are of the same order. Recovery of the correct periods was also possible by the fact that the bulk of the data was observed when the phase angle changes were small. This explains why we can derive the component periods of the rotational state of Toutatis from the entire lightcurve despite the changes in the Earth-Toutatis-sun geometry during the December/January observing period.

4 Conclusion

Our primary conclusion is that with appropriate tools for determining periodicities in a lightcurve and with good temporal coverage, important aspects of the complex rotation state of an elongated asteroid can be diagnosed with high precision. The frequencies associated with precession of the long axis about the angular momentum vector and the period of rotation or oscillation about the long axis can be determined. It is not always possible, however, to distinguish between spin in a SAM or LAM mode on the basis of the periodicities alone. In the case of Toutatis we can eliminate a SAM state because \( P_\psi < P_\varphi \). To fully define the rotation state, information on the ratios of principal moments of inertia (or accurate axial lengths) and the orientation of the rotational angular momentum vector
Figure 9: Evolution of the sub-solar (solid line) and sub-Earth (dashed line) directions during the December/January observations. The solid squares and open circles denote sub-solar and sub-Earth directions every 13 days during the December/January observations. The sub-solar direction is moving approximately in the direction opposite to the changing sub-Earth direction. The arrows denote the flow of time.

is required. To some extent, in cases with favorable geometry, some of this information might be derived from lightcurve amplitudes and, in the case of active comets, from the time development of features in their comae.

Large changes of the orientation/observing geometry during the observed time interval is detrimental to the recovery of the correct periodicities, but if the bulk of the data corresponds to an approximately constant aspect (as was the case for Toutatis) and the lightcurve covers an interval much larger than the component periods, then we can still recover the periodicities accurately.

WindowCLEAN is a powerful tool for periodicity analysis of lightcurves. In future work we will expand our model simulations and subsequent WindowCLEAN analysis to widely different shape models and will apply our FT techniques to other observed lightcurves of asteroids with suspected complex spin.

Combining the periodicity analysis of visible lightcurves with WindowCLEAN with the shape inversion of radar observations maybe a powerful tool to deduce non-principal axis spin states by significantly reducing the spin state parameter space that the shape inversion algorithm has to search.

This paper is the first definite derivation of component periods of a complex spin state
from a visible lightcurve. Our work is a completely independent confirmation of the spin state deduced by Hudson and Ostro (1995) as well as of the photometric parameters derived in Hudson and Ostro (1998).

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