Technological Challenges

HST - 1990

Space-Based Observatory
Multipurpose UV/Visual/IR Imaging and Spectroscopy

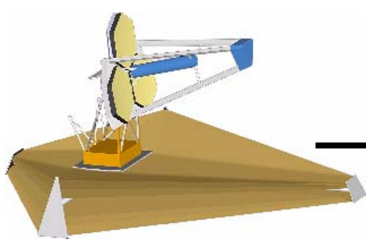
The next generation of space based observatories is expected to provide significant improvements in angular resolution, spectral resolution and sensitivity.

Science Requirements → Engineering Requirements → D&C System Requirements

5 year wide-angle astro-metric accuracy of 4 μasec to limit 20th Magnitude stars → Fringe Visibility > 0.8 for astrometry → Science Interferometer OPD < 10 nm RMS

Sample Requirements Flowdown for SIM

NEXUS-2004



Deployable Cold Optics
NGST Precursor Mission

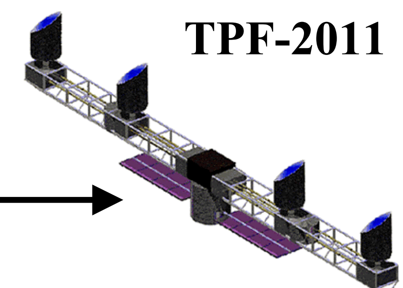
NGST-2009

Lightweight 8m-Optics
IR Deep Field Observations

SIM-2006

Faint Star Interferometer
Precision Astrometry

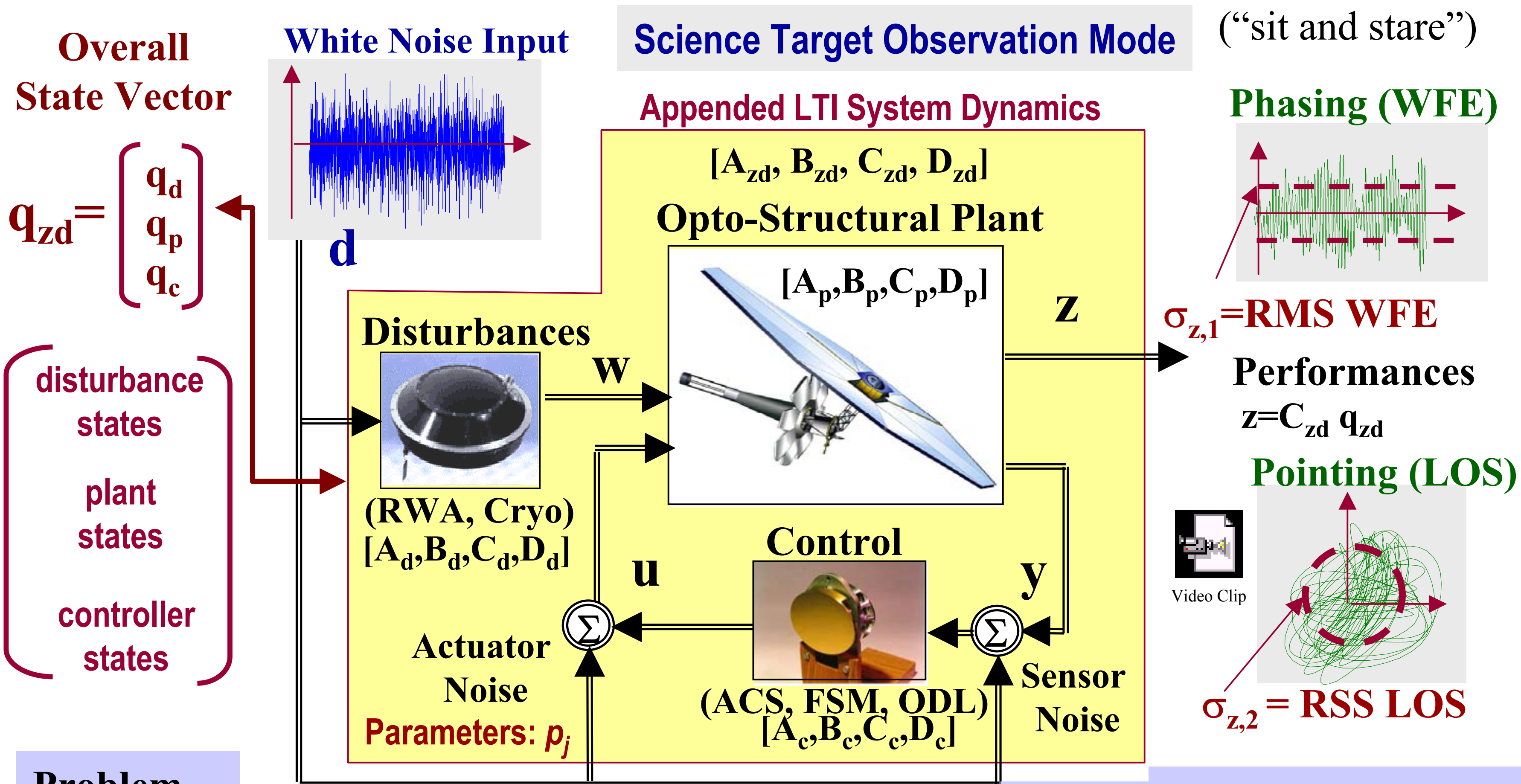
TPF-2011



Nulling Interferometer
Planet Detection

Achieve requirements in a cost-effective manner with predictable risk level.

Research Motivation - Problem Statement



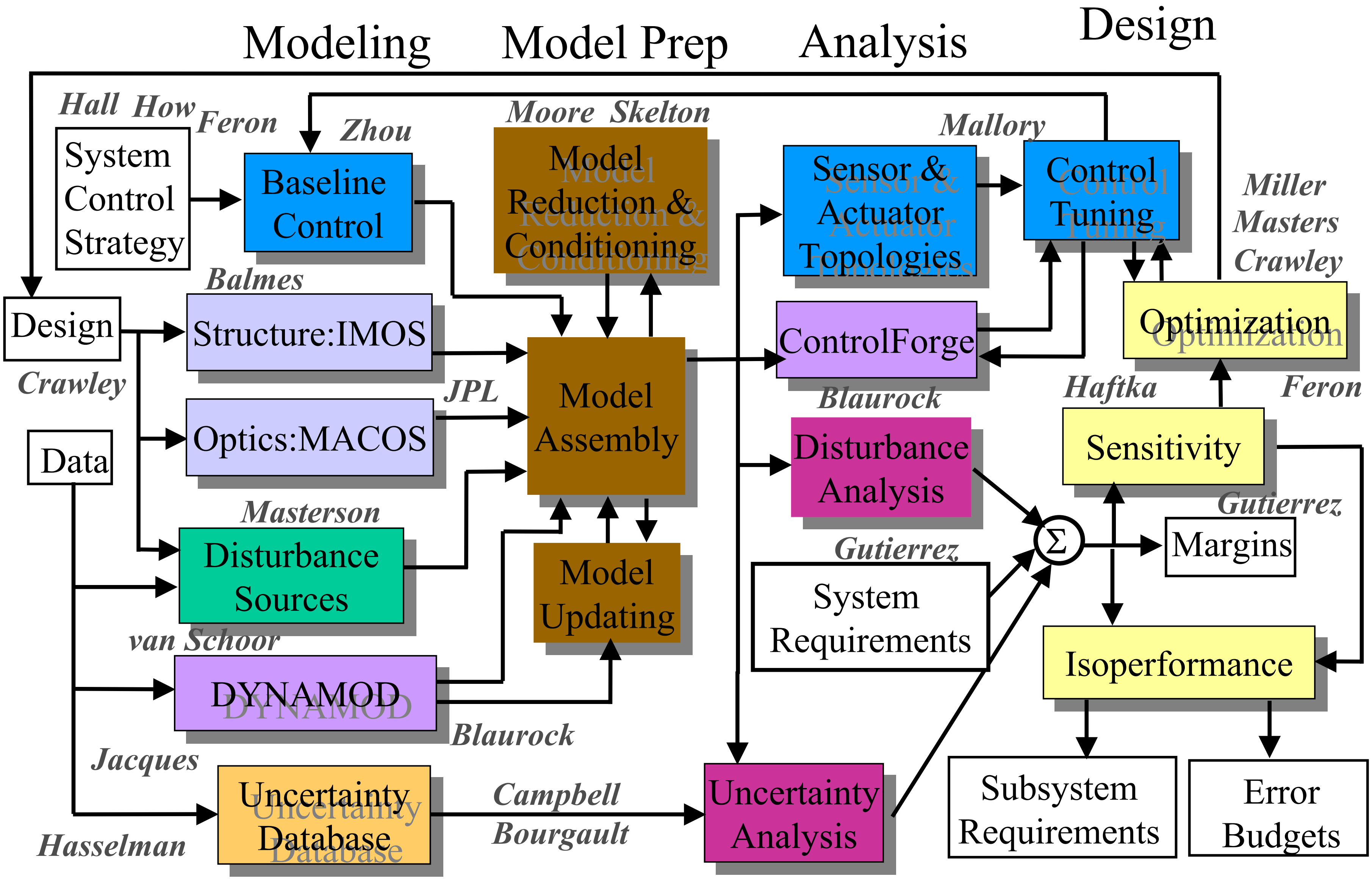
Problem Statement:

$n_p > n_z$ # of parameters: $j = 1 \ldots n_p$
# of performances: $i = 1 \ldots n_z$

Traditionally: Define System Parameters $p_j = p_o$ → Predict H_2 performances $\sigma_{z,i}$

Isoperformance: Find Loci of Solutions $p_{LB} < p_j < p_{UB}$ ← Constrain performances $\sigma_{z,i} = \sigma_{z,req}$

Dynamics-Optics-Controls-Structures Framework



Current Evaluation Framework

Using Lyapunov Approach:

1

$$\dot{q} = A_{zd} q + B_{zd} d$$
$$z = C_{zd} q$$

Steady-State Lyapunov Equation

2

$$A_{zd}\Sigma_q + \Sigma_q A_{zd}^T + B_{zd} B_{zd}^T = 0$$

RMS

3

$$\sigma_{z_i} = \left(C_{zd,i} \Sigma_q C_{zd,i}^{\ T} \right)^{1/2}$$

Sensitivity

6

$$\frac{\partial \sigma_{z_i}}{\partial p_j} = \frac{1}{2\sigma_{z_i}} \cdot \frac{\partial \sigma_{z_i}^2}{\partial p_j}$$

Governing Sensitivity Equation (GSE)

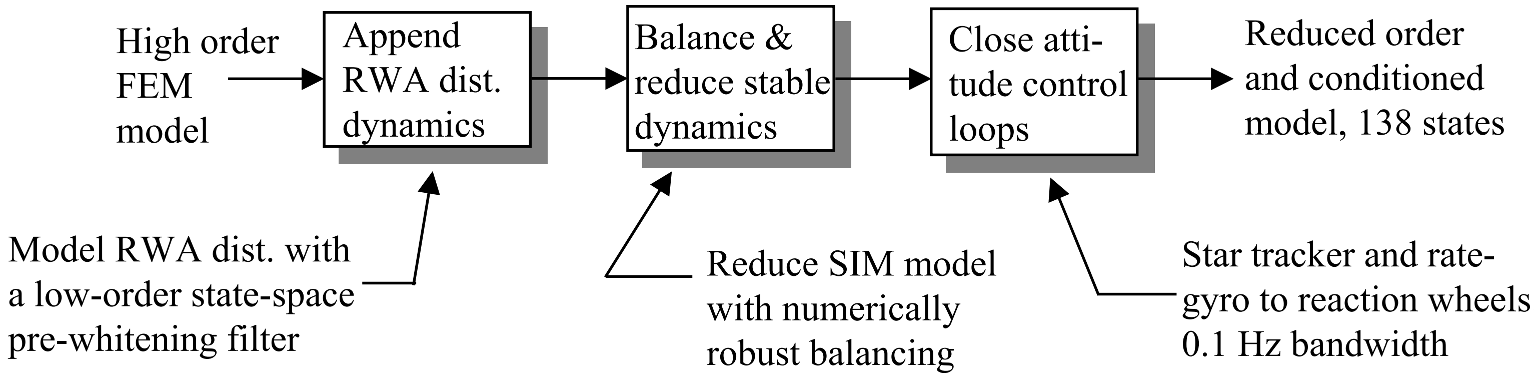
5

$$\frac{\partial \sigma_{z_i}^2}{\partial p_j} = tr\left[\Sigma_q \frac{\partial \left(C_{zd,i}^T C_{zd,i} \right)}{\partial p_j} \right] + tr\left[L_i \left\{ \frac{\partial A_{zd}}{\partial p_j} \Sigma_q + \Sigma_q \frac{\partial A_{zd}^T}{\partial p_j} + \frac{\partial \left(B_{zd} B_{zd}^T \right)}{\partial p_j} \right\} \right]$$

4 Lagrange Multiplier Matrix Equation

$$L_i A_{zd} + A_{zd}^T L_i + C_{zd,i}^T C_{zd,i} = 0$$

Model Assembly and conditioning



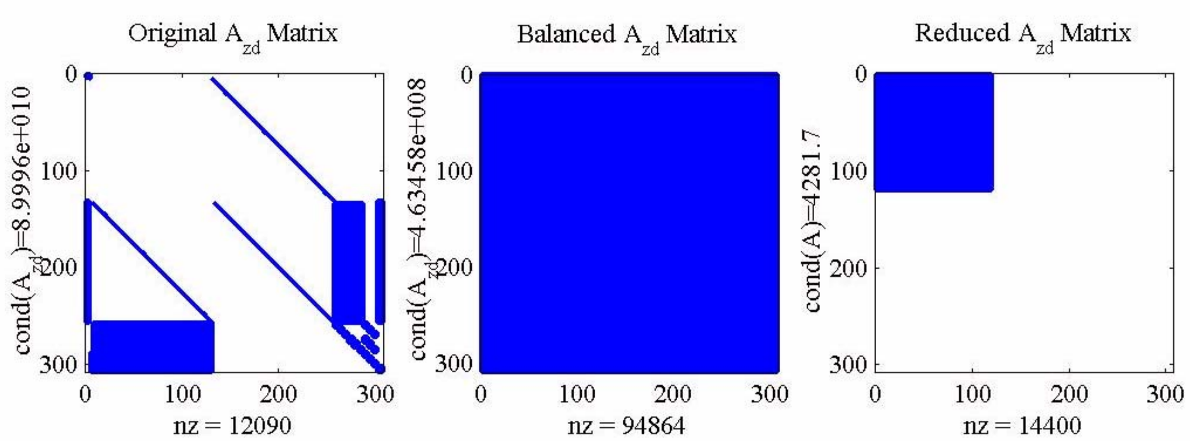
Create appended dynamic LTI system:

$$A_{zd} = \begin{bmatrix} A_d & 0 & 0 \\ \begin{bmatrix} B_w \\ B_c D_{yw} \end{bmatrix} C_d & \begin{bmatrix} A_p & B_u C_c \\ B_c C_y & A_c + B_c D_{yu} C_c \end{bmatrix} \end{bmatrix}$$

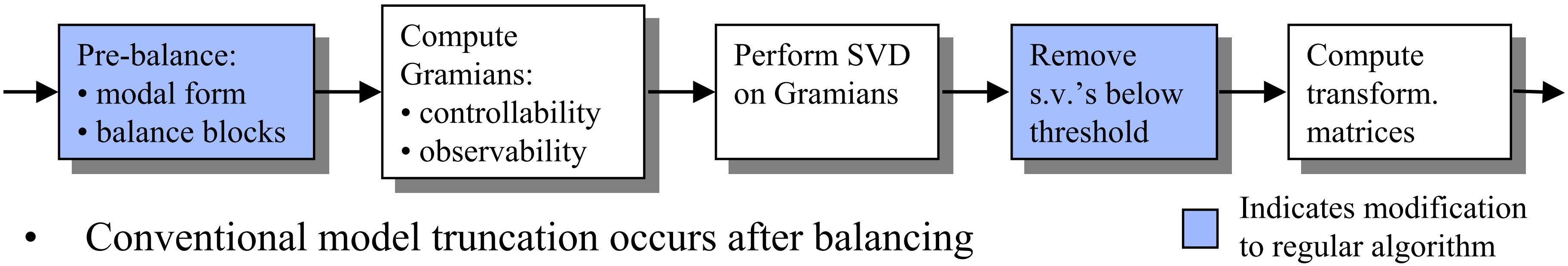
SIM Classic

308 States (Full Order)

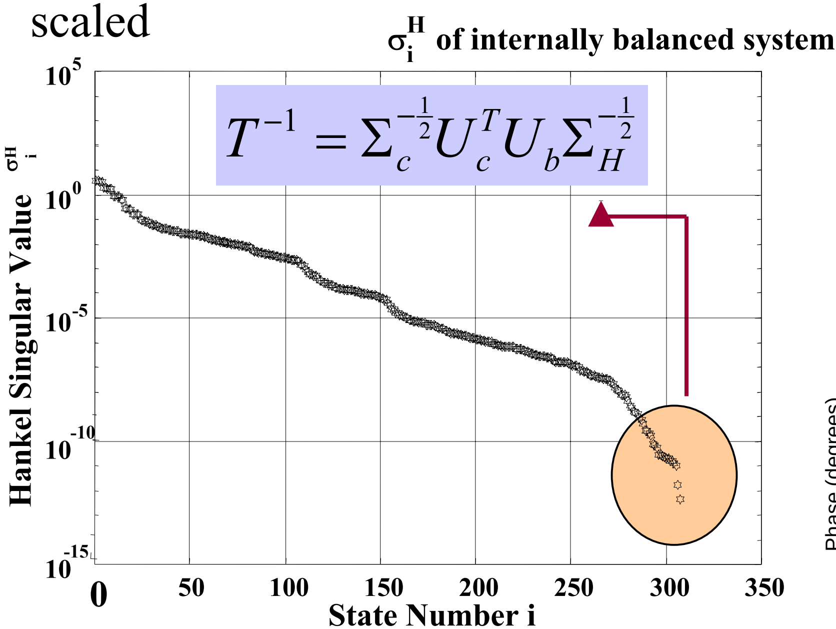
110 States (Balanced Reduced)

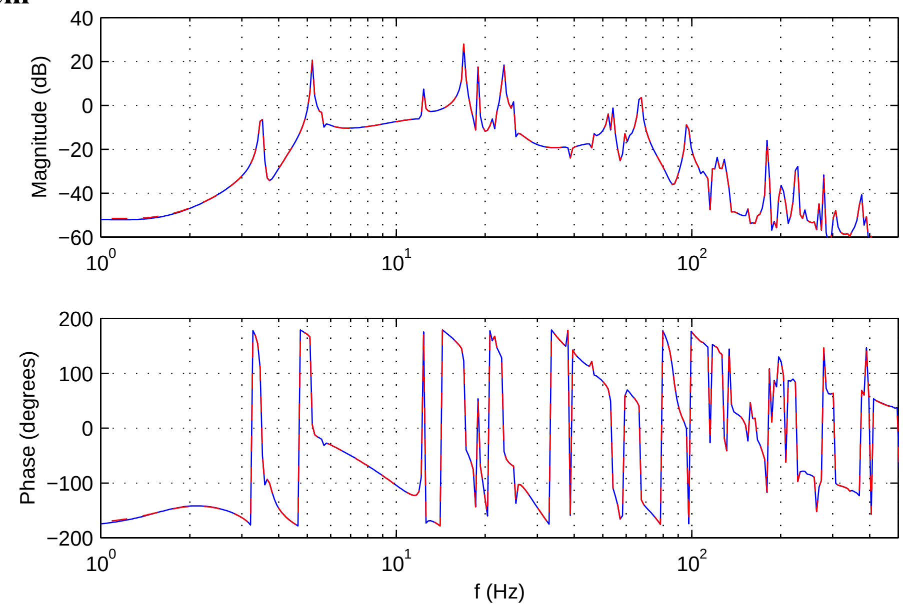


Numerically Robust Balancing Algorithm

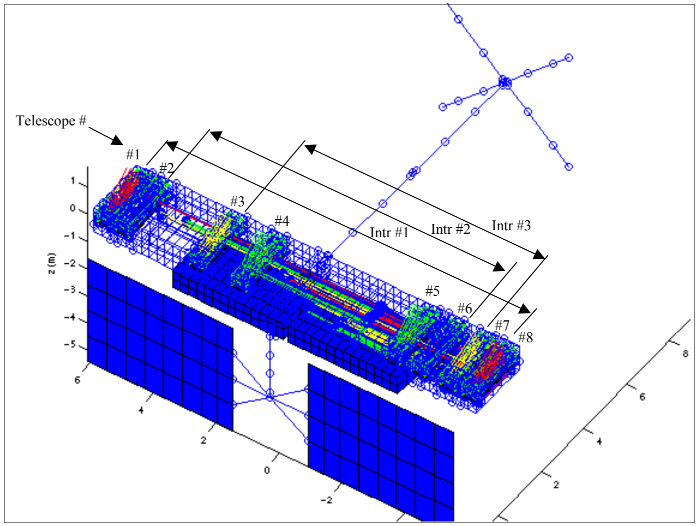


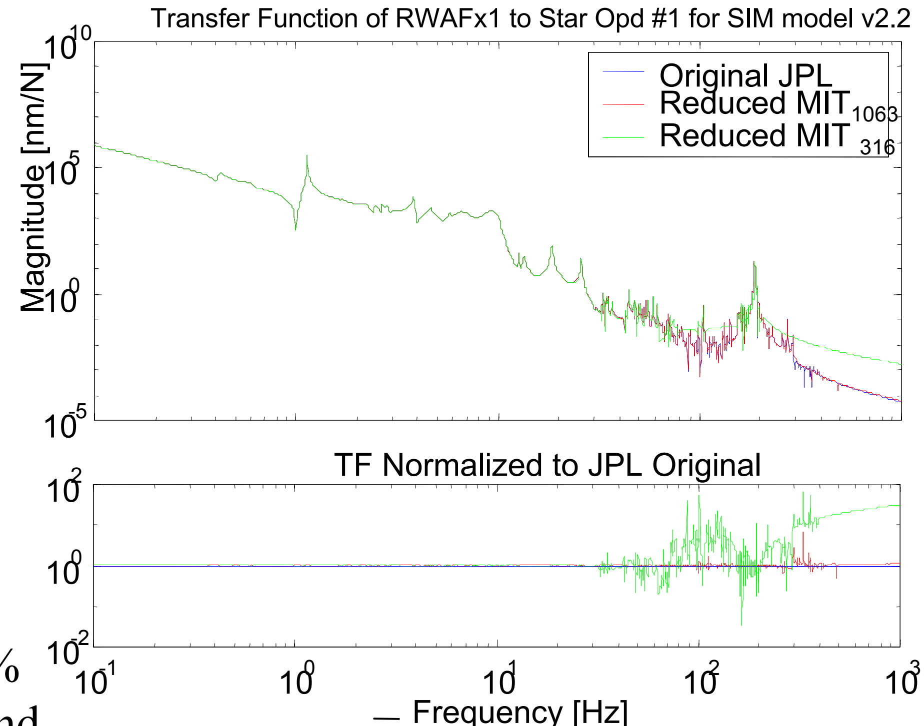
- Conventional model truncation occurs after balancing
- Modified Algorithm: truncation occurs during balancing, controlled by threshold
- Pre-balancing ensures that 2x2 blocks corresponding to each mode are acceptably scaled





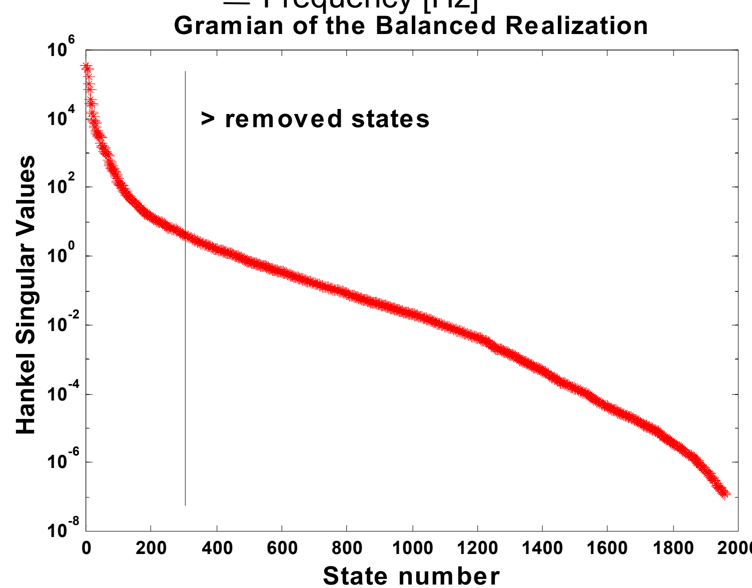
SIM Model Reduction





Select number of states to retain based on % RMS difference between reduced system and full-order system

$$\frac{\Delta\sigma_{z_i}}{\bar{\sigma}_{z_i}} < \frac{1}{2}\frac{\sum_{i=k+1}^{n}\sigma_i^H}{\sum_{i=1}^{k}\sigma_i^H} \qquad \sigma_i^H = \text{i}^{\text{th}}\, H.S.V.$$



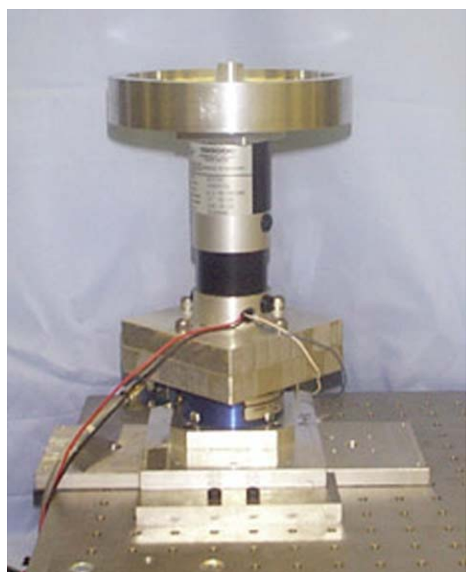
0.01% difference → 316 retained states

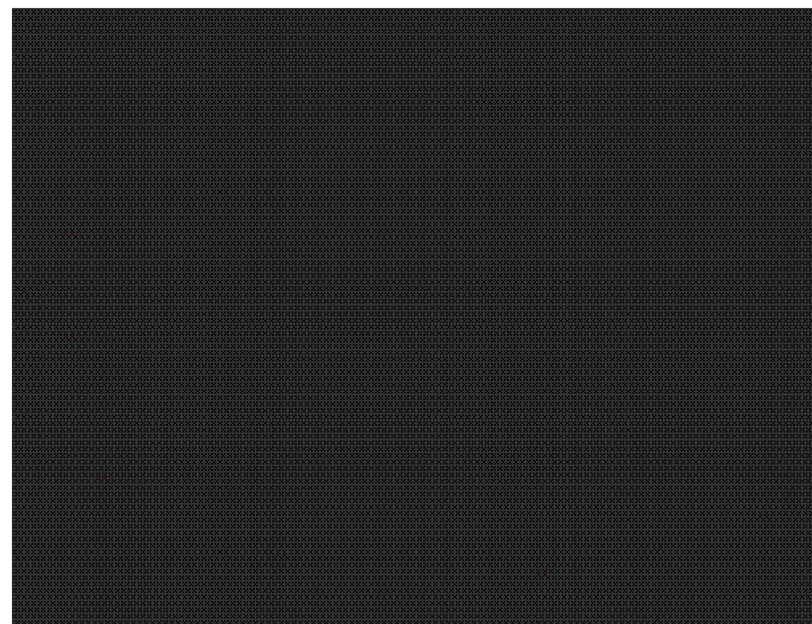
Keep more to match original T.F. more accurately

RWA Testing & Modeling

- Reaction wheels are anticipated to be largest source of disturbances for Precision Space Structures
- Static, dynamic imbalances induce disturbance at freq. of wheel spin
- Bearing, motor, dynamic lubricant disturbances induce vibration at higher (and sub) harmonics of wheel speed
- Experiment & empirical modeling
- Analytical modeling

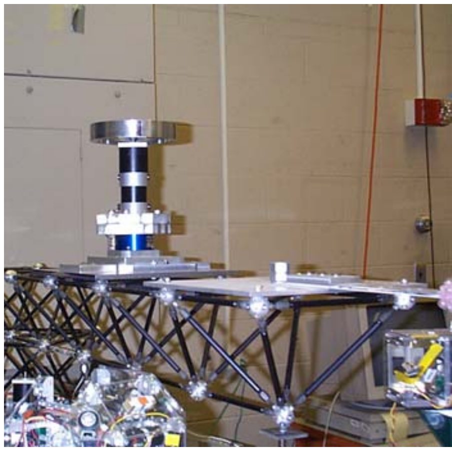
Ithaco “B” RW

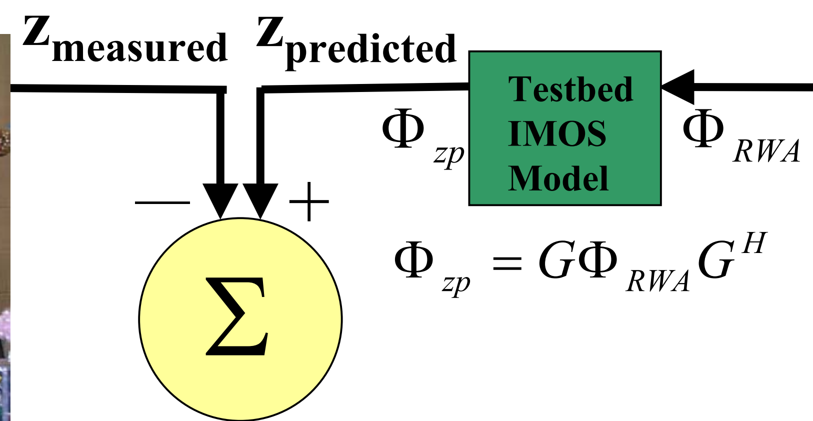




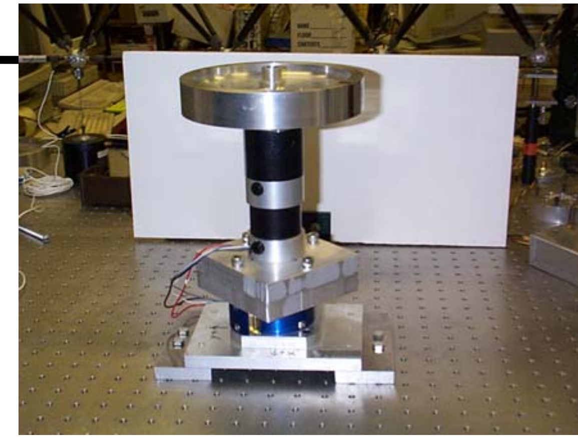
Impedance Coupling Analysis

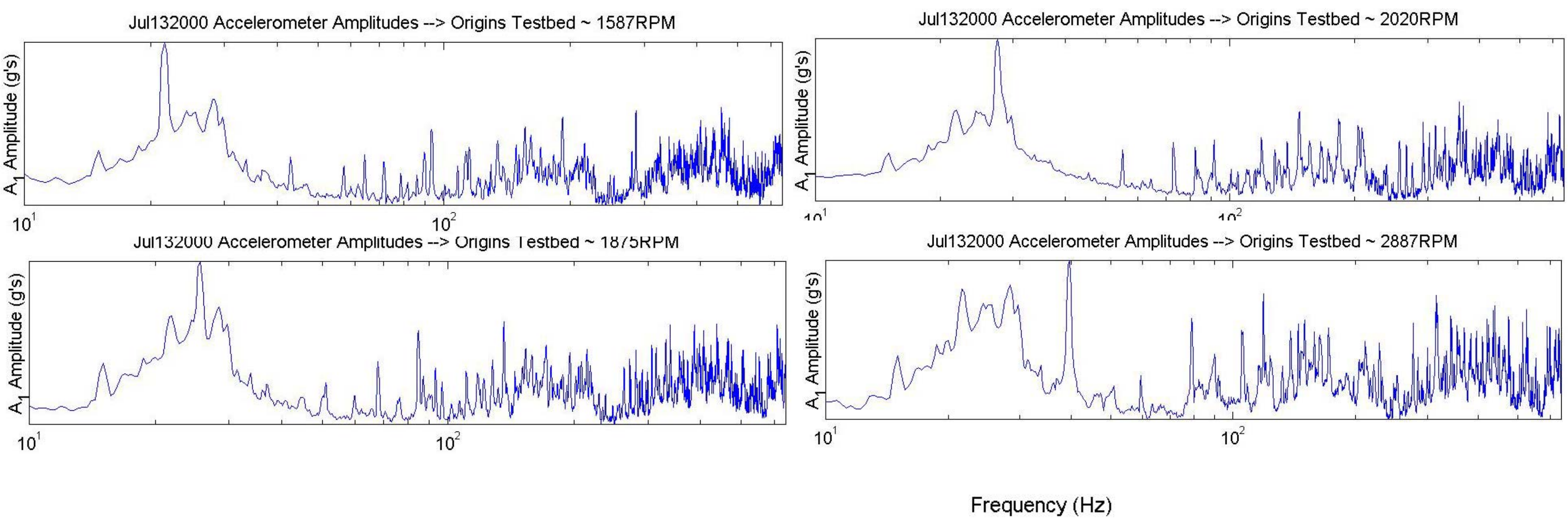
Coupled



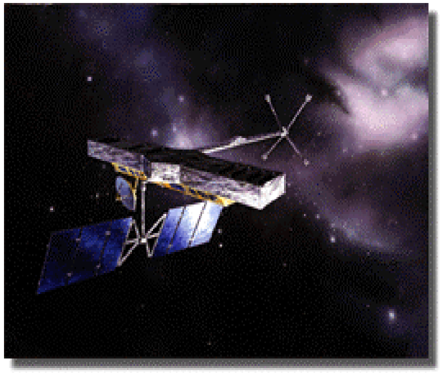


Hardmounted





Disturbance Analysis



Performed a **disturbance analysis** and **modal parameter sensitivity analysis** on closed-loop SIM Classic Model to validate framework

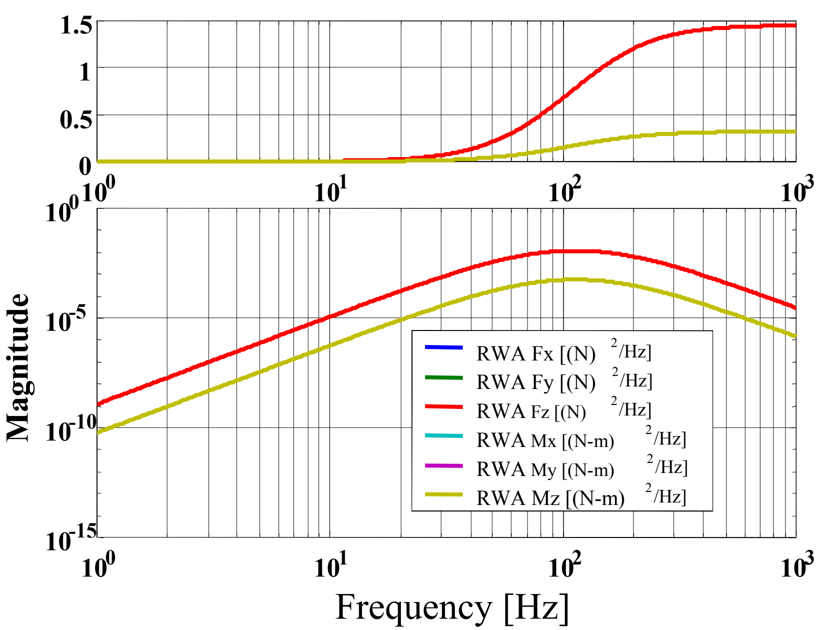
Disturbance (left): Reaction Wheel Assembly
Performance (right): Total OPD (int. #1)

Comparison

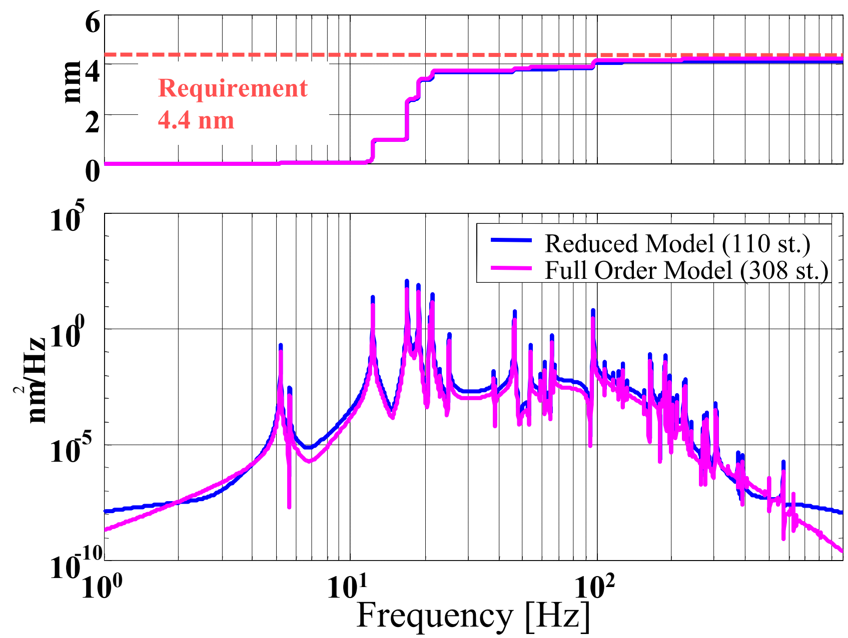
Results	Full Model	Red Model
# States	308	110
RMS (PSD)	4.21 nm	4.21 nm
RMS (Lyap)	4.3321 nm	4.1077 nm
CPU (Lyap)	39.567 sec	1.552 sec

Nominal RMS error < 0.2 %

Disturbance PSD's and cumulative RMS curves



SIM Classic Total OPD (int. #1) Power Spectral Density



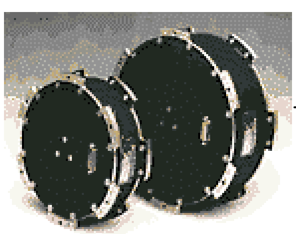
SIM Broadband Disturbance Analysis

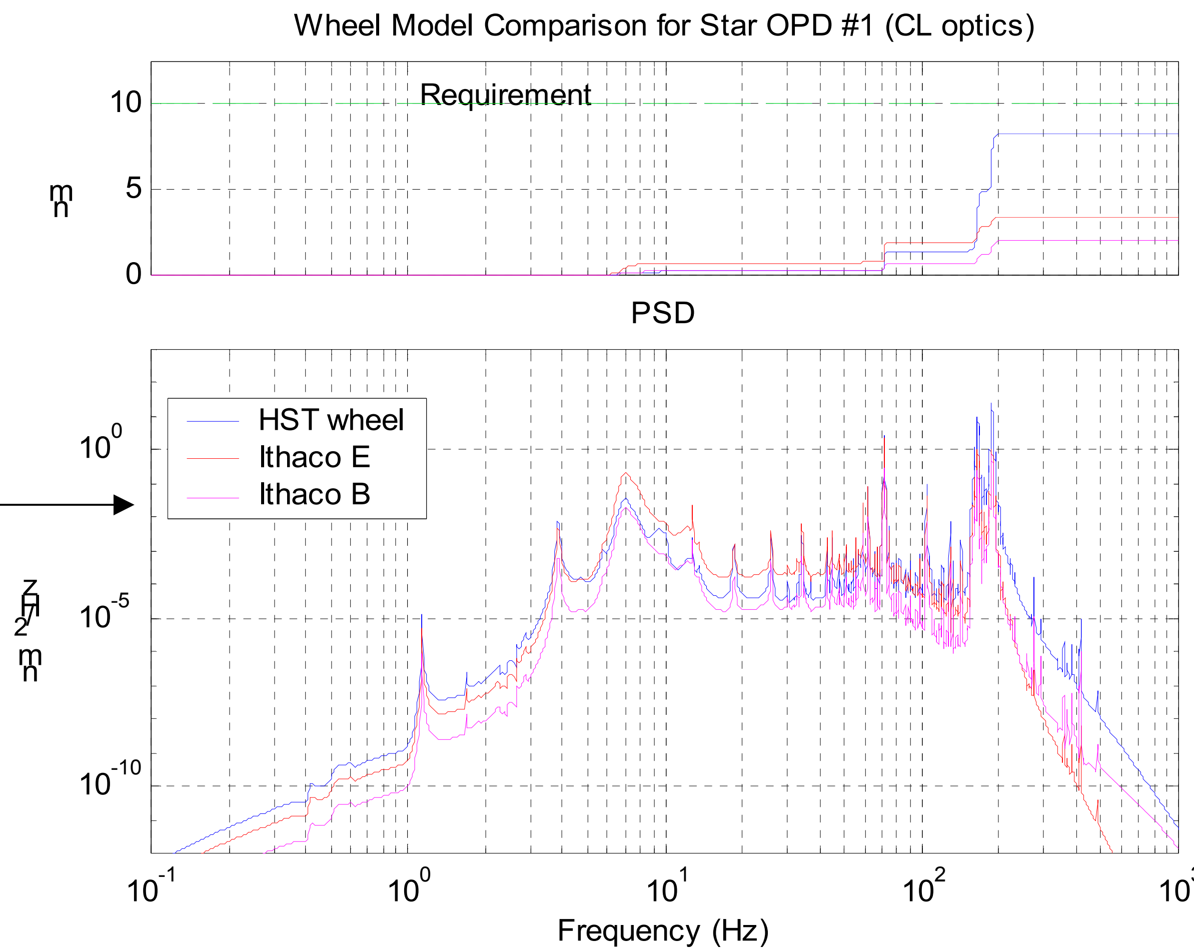
Question: How does the **choice of reaction wheel** (disturbance model) affect the broadband disturbance analysis results ?

HST Wheel*

ITHACO E

ITHACO B





* actually represents Honeywell HR2020 wheel

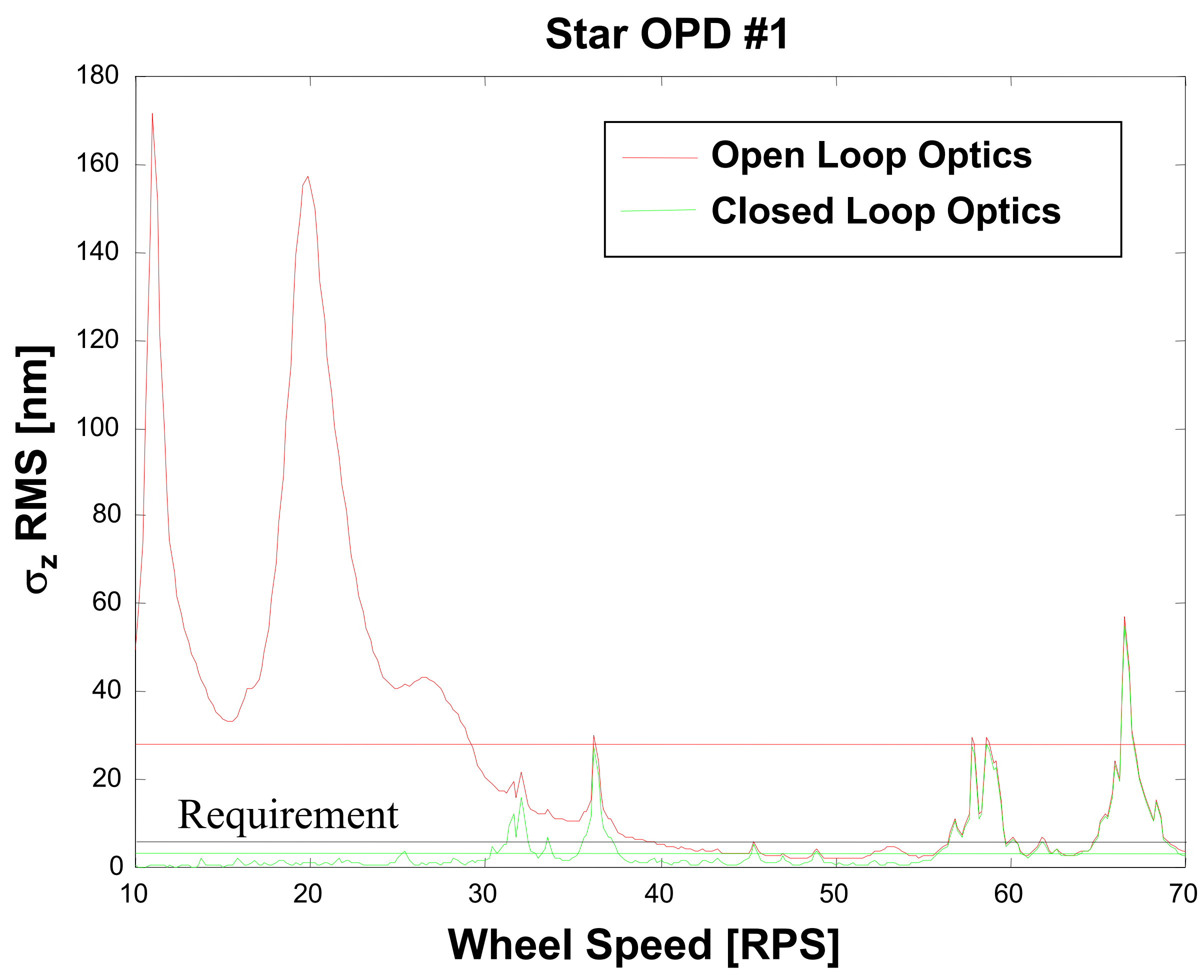
Note: Teldix Wheel information not released to MIT

SIM Disturbance Analysis Results
Discrete - Star OPD #1

Plot shows total performance RMS at each wheel speed.

Closed loop performances improve at low wheels speeds (< 30 RPS); not affected at higher wheel speeds.

Disturbance contributions from higher harmonics at high wheel speeds are above the optical bandwidth (100 Hz).



Average values represented by dashed lines.

Requirement represented by black line.

Lyapunov Block Diagonal Solutions

$$[0] = \begin{bmatrix} A_1 & & & & 0 \\ & A_2 & & & \\ & & A_3 & & \\ & & & \dots & \\ 0 & & & & A_n \end{bmatrix} \begin{bmatrix} X_{11} & X_{12} & \dots \\ X_{21} & X_{22} & \\ \dots & & \dots \end{bmatrix} +$$

$$\begin{bmatrix} X_{11} & X_{12} & \dots \\ X_{21} & X_{22} & \\ \dots & & \dots \end{bmatrix} \begin{bmatrix} A_1^T & & & & 0 \\ & A_2^T & & & \\ & & A_3^T & & \\ & & & \dots & \\ 0 & & & & A_n^T \end{bmatrix} + \begin{bmatrix} BB_{11} & BB_{12} & \dots \\ BB_{21} & BB_{22} & \\ \dots & & \dots \end{bmatrix}$$

$$A_1 X_{11} + X_{11} A_1^T + BB_{11} = 0 \qquad A_1 X_{12} + X_{12} A_2^T + BB_{12} = 0$$

$$A_2 X_{21} + X_{21} A_1^T + BB_{21} = 0 \qquad \dots.$$

Computation Times of New Gramian Solver

- Decouple the full Lyapunov equation (solving for a nxn matrix) into $(n^2+n)/2$ separate Lyapunov equations, each solving for a mxm (block size) matrix.
- Trade-off between faster **`lyap.m`** computations for small block size and **`for`**…**`end`** loop losses.
- Improvement in computation time tested with SIM model in 2x2 block diagonal, 2nd order form.
- Dramatic improvement in time; equal accuracy. Note odd block sizes fail.

Method	Time [sec]	Max. Resultant
`lyap.m` on full n *x* n	169.14	$2.55*10^{-15}$
m=2	33.29	$2.55*10^{-15}$
m=23	Solution does not exist	
m=46	3.75	$2.55*10^{-15}$
m=47	Solution does not exist	
m=94	5.15	$2.55*10^{-15}$

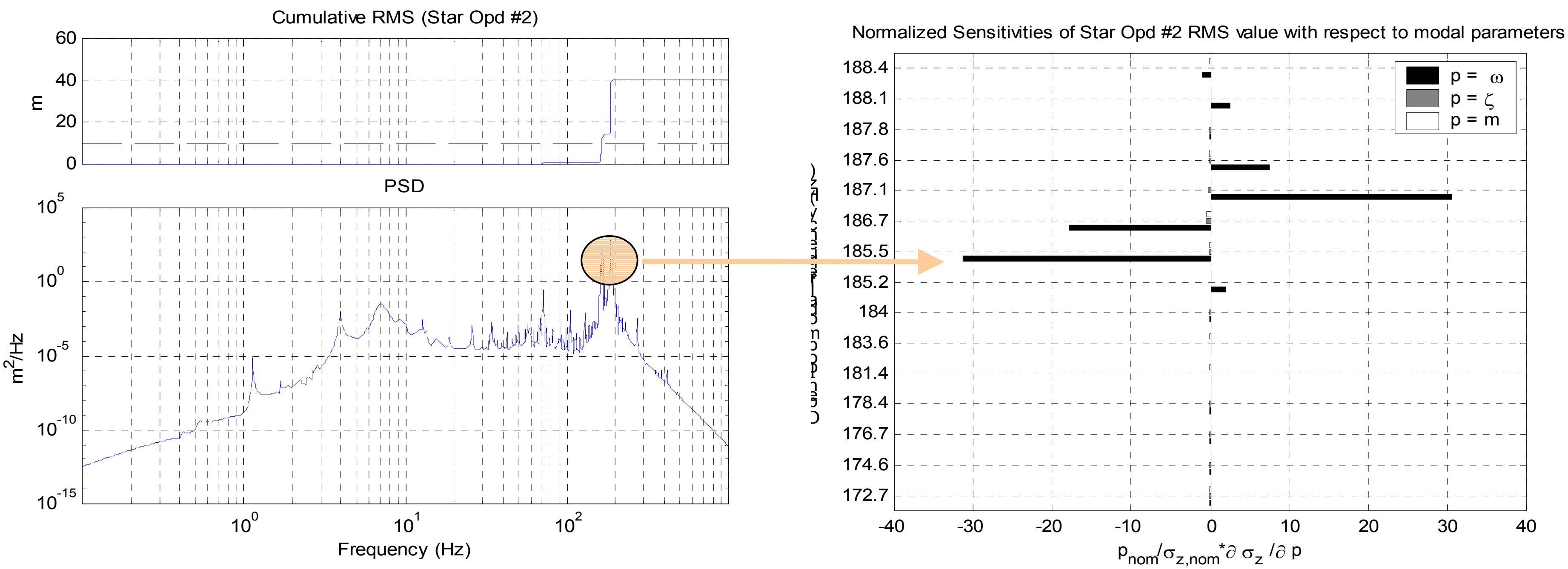
SIM Modal Sensitivity Analysis

Find sensitivity (partial derivatives) of the performances with respect to modal or physical system parameters.

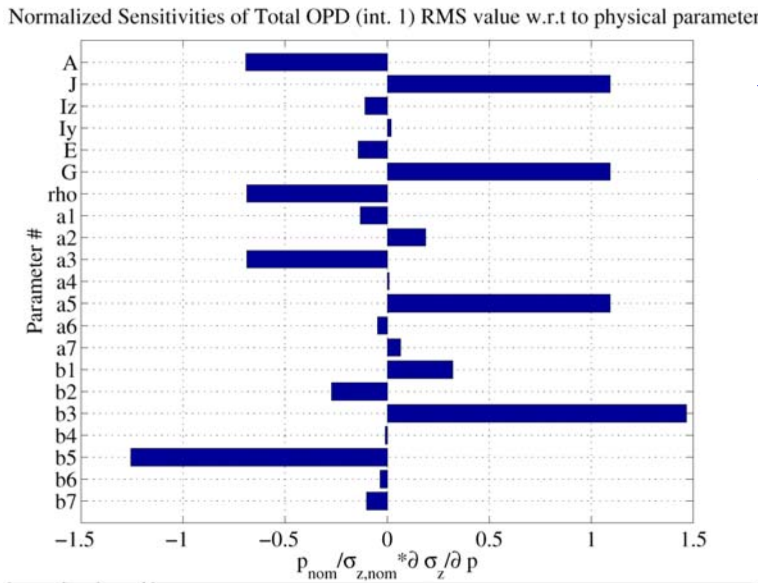
$$\frac{\partial \sigma_{z_i}}{\partial p_j} = \frac{1}{2\sigma_{z_i}} \cdot \frac{\partial \sigma_{z_i}^2}{\partial p_j}$$

Governing Sensitivity Equation (GSE)

$$\frac{\partial \sigma_{z_i}^2}{\partial p_j} = tr\left[\Sigma_q \frac{\partial\left(C_{zd,i}^T C_{zd,i}\right)}{\partial p_j}\right] + tr\left[L_i\left\{\frac{\partial A_{zd}}{\partial p_j}\Sigma_q + \Sigma_q \frac{\partial A_{zd}^T}{\partial p_j} + \frac{\partial\left(B_{zd}B_{zd}^T\right)}{\partial p_j}\right\}\right]$$



Physical Parameter Sensitivities



A, J, I_z, I_y, E, G, ρ: siderostat boom properties
a1-a7: α scale factors for siderostat CELAS's
b1-b7: β scale factors for siderostat CONM's

Physical Insight:
G and **J** are the most important physical parameters for the siderostat boom. Also **a3/b3** indicate that local siderostat modes affect performance.

Physical Parameter Sensitivities can be obtained in 2 ways:

(1) Modal method (via chain rule):
Sum only over important DOF's and modes that are kept in the reduced model.

OR

(2) Direct method (in physical space)

$$\frac{\partial A_{zd}}{\partial p} = \sum_{j=1}^{N} \left(\frac{\partial A_{zd}}{\partial \omega_j} \cdot \frac{\partial \omega_j}{\partial p} \right)$$

$$\frac{\partial B_{zd}}{\partial p} = \sum_{j=1}^{N} \left(\frac{\partial B_{zd}}{\partial \hat{m}_j} \cdot \frac{\partial \hat{m}_j}{\partial p} + \sum_{i=1}^{n_m} \left(\frac{\partial B_{zd}}{\partial {}^o\phi_{ij}} \cdot \frac{\partial {}^o\phi_{ij}}{\partial p} \right) \right)$$

$$\frac{\partial C_{zd}}{\partial p} = \sum_{j=1}^{N} \sum_{i=1}^{n_m} \left(\frac{\partial C_{zd}}{\partial {}^o\phi_{ij}} \cdot \frac{\partial {}^o\phi_{ij}}{\partial p} \right)$$

Matrix Derivatives for a "Structural Plant-only" System.

Only sum over open loop modes that are kept.

Only sum over "important" deg. of freedom

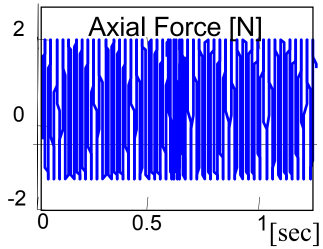
Dynamics and Controls Error Budgeting (1)

(1) Why is error budgeting important ?	Establishes feasibility of dynamic system performance given noise source assumptions.
(2) How is it done today?	Ad-Hoc error budgeting, RSS error tree, limited physical understanding of interactions.
(3) How can isoperformance help error budgeting ?	Leverages sensitivity analysis and integrated modeling. Creates **link to physical parameters**.

Goal: Balance anticipated error sources, which are given by physical process limits and imperfections of hardware in a predictable and physically realizable manner. Example: balancing of sensor vs. process noise.

NGST Example : Assume 3 Main Error Sources

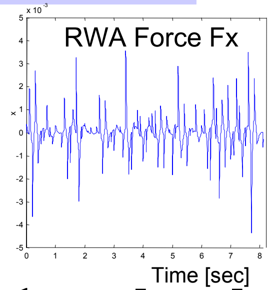
Error Source 1: CRYO



Q_c: Amplification Factor [-]

$0.005 <= Q_c <= 0.05$

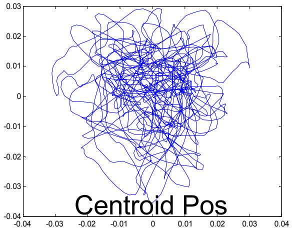
Error Source 2: RWA



U_s: Static Imbalance [gcm]

$1.0 <= U_s <= 30.0$

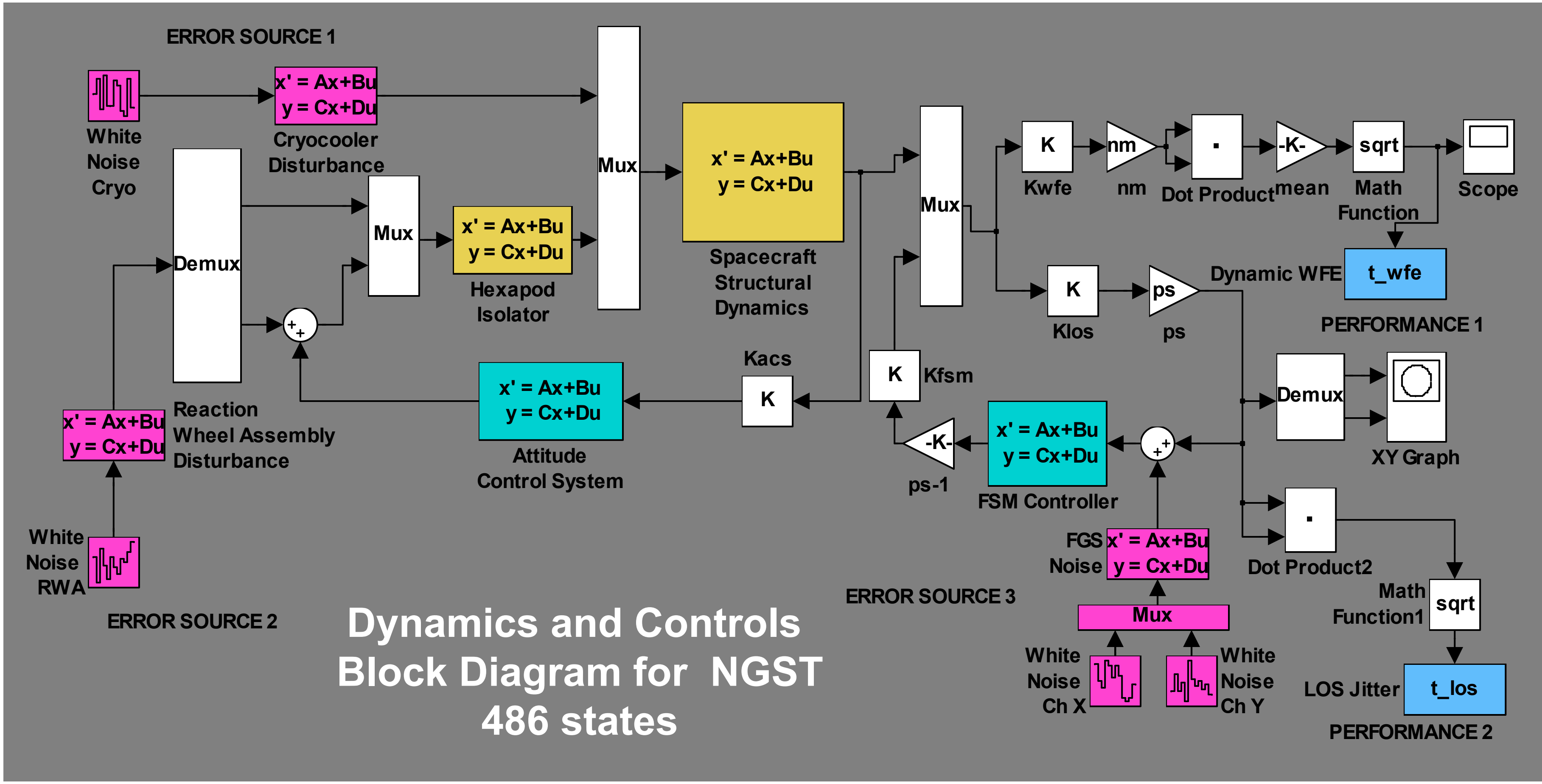
Error Source 3: GS Noise



T_{int}: Integration Time [sec]

$0.020 <= T_{int} <= 0.100$

Dynamics and Controls Error Budgeting (2)



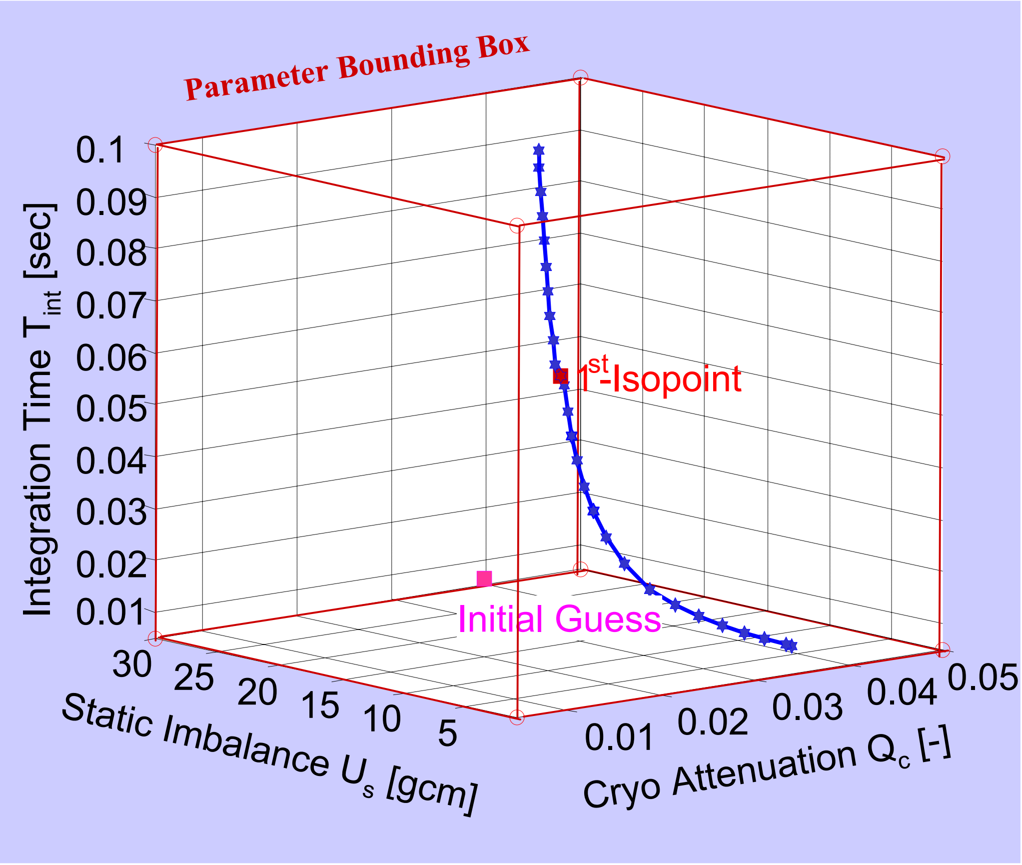
Disturbance Parameters (Inhomogeneous Dynamics) (variable)

Plant Parameters (Homogeneous Dynamics) (fixed)

Control Parameters (Homogeneous Dynamics) (fixed)

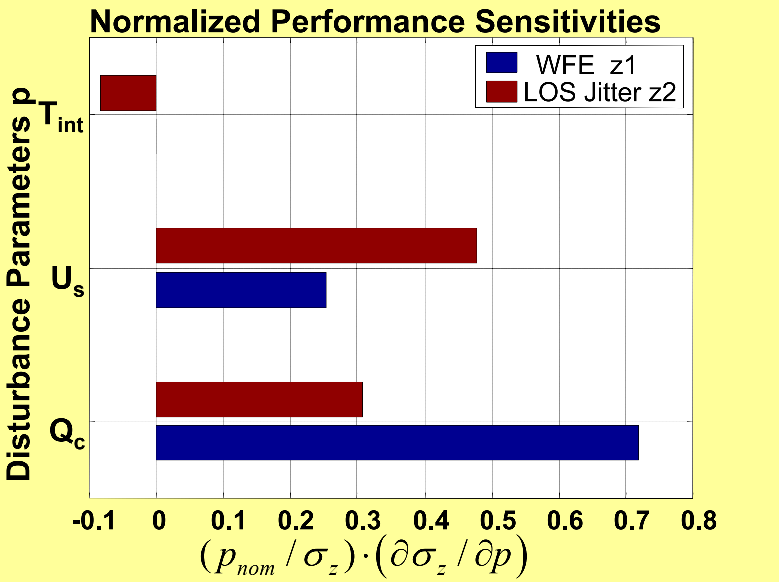
Dynamics and Controls Error Budgeting (3)

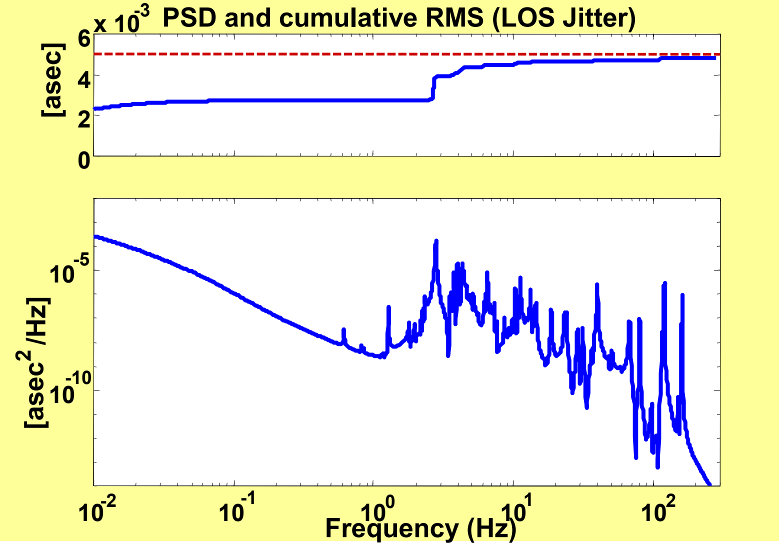
Isoperformance Analysis Results



Isocontour for Performances:

$\sigma_{z,\text{ WFE}} = 55$ [nm] ; $\sigma_{z,\text{LOS}} = 0.005$ [asec]

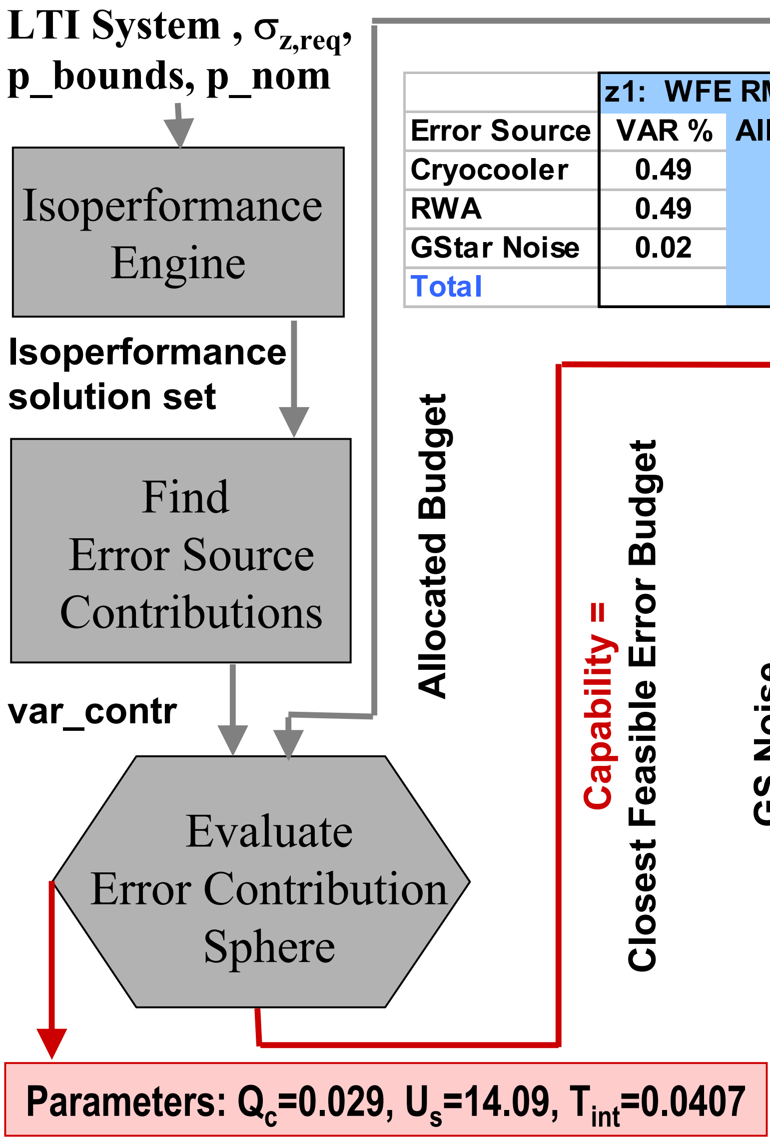


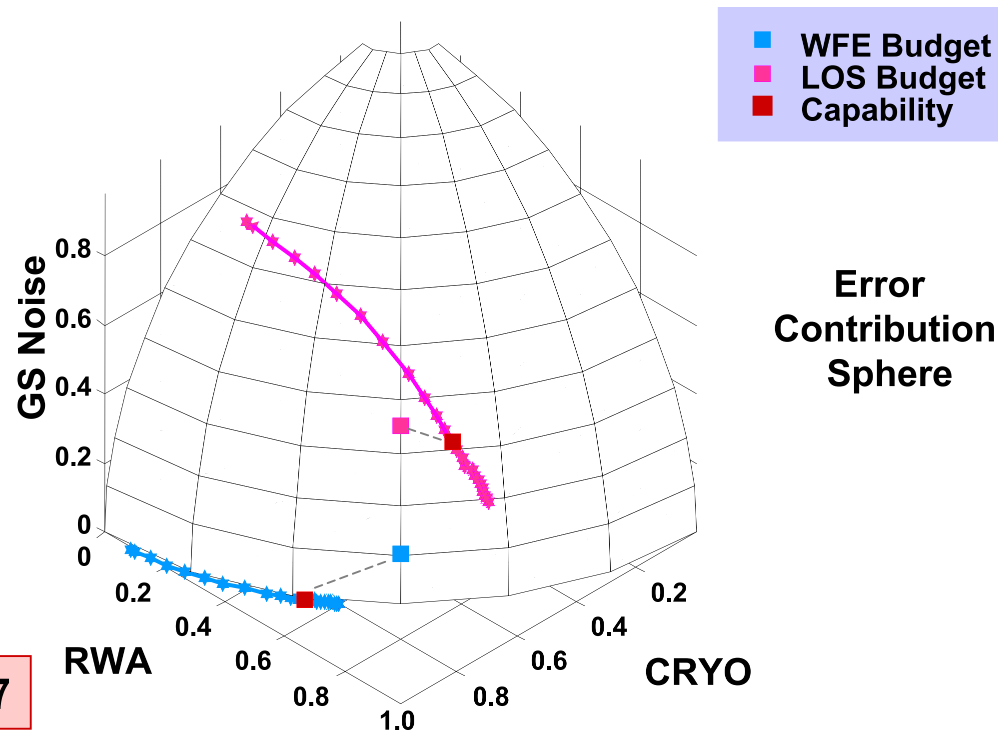


Dynamics and Controls Error Budgeting (4)

Example: NGST Error Budget (Excel)

	z1: WFE RMS [nm]		Req:	55.00	z2: LOS Jitter [asec]		Req:	0.005
Error Source	VAR %	Allocation	VAR%	Capability	VAR %	Allocation	VAR%	Capability
Cryocooler	0.49	38.50	0.72	46.6467	0.40	0.003162	0.31	0.002823
RWA	0.49	38.50	0.28	29.1543	0.40	0.003162	0.53	0.0037016
GStar Noise	0.02	7.78	0.00	0.0000	0.20	0.002236	0.17	0.0020829
Total		55.00		55.0081		0.005000		0.0051





(Bivariate) Isoperformance Fundamentals

k-th isopoint:

$$p_k = \begin{bmatrix} p_{1,k} \\ p_{2,k} \end{bmatrix}$$

Vector function: $p_k \mapsto \sigma_z(p_k), \; where \; \mathbb{R}^2 \mapsto \mathbb{R}$

Taylor series expansion :

$$\sigma_z(p) = \sigma_z(p_k) + \underbrace{\nabla \sigma_z^T \Big|_{p_k} \Delta p}_{\text{first order term}} + \underbrace{\frac{1}{2} \Delta p^T \, H \Big|_{p_k} \Delta p}_{\text{second order term}} + \text{HOT}$$

H: Hessian

$$\nabla \sigma_z^T \Big|_{p_k} \Delta p \equiv 0$$

$$U_k S_k V_k^T = \nabla \sigma_z^T \Big|_{p_k}$$

$$V_k = \left[n_k \quad t_k \right]$$

t_k: Tangent vector is in the nullspace

n_k: Normal Vector

α_k : Step size
t_k : Step direction

$$\alpha_k = \left[2 \frac{\varepsilon_{iso} \sigma_{z,req}}{100} \left(t_k^T \, H \Big|_{p_k} t_k \right)^{-1} \right]^{1/2}$$

$$\Delta p = \alpha_k \cdot t_k$$

$$p_{k+1} = p_k + \Delta p$$

P_{k+1} : k+1th isopoint, where k=1,…,n_{iso}

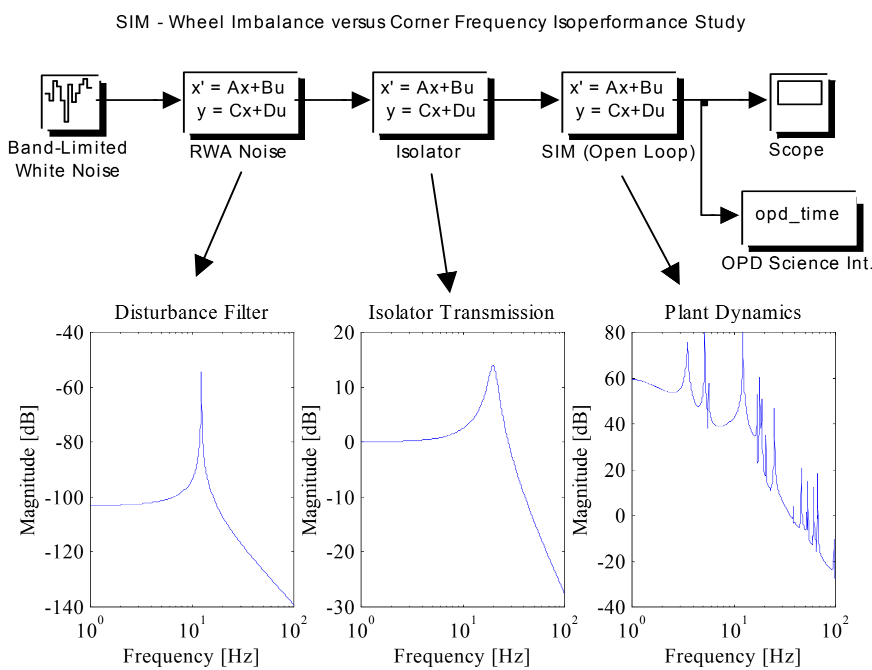
Quality of isoperformance solution

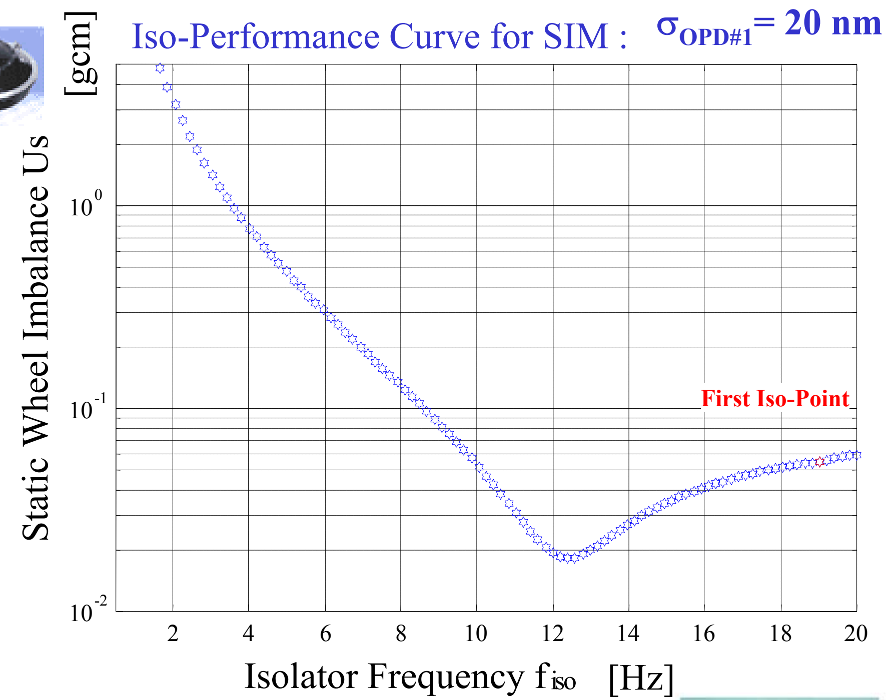
$$\Upsilon_{iso} = \frac{100}{\sigma_{z,req}} \cdot \left[\frac{\sum_{k=1}^{n_{iso}} \left[\sigma_{z,k} - \bar{\sigma}_z \right]^2}{n_{iso}} \right]^{1/2}$$

Isoperformance for SIM (1)

Isoperformance analysis for static wheel imbalance U_s [gcm] versus isolator corner frequency f_{iso} [Hz] at the RMS OPD #1 = 20 nm level for SIM Classic (version 1.0)

$$\Delta\sigma_z = \frac{\partial\sigma_z}{\partial U_s}\Delta U_s + \frac{\partial\sigma_z}{\partial f_{iso}}\Delta f_{iso} = 0$$





Observation:

“Dip” in isoperformance contour corresponds to region, where isolator amplifies the disturbance.

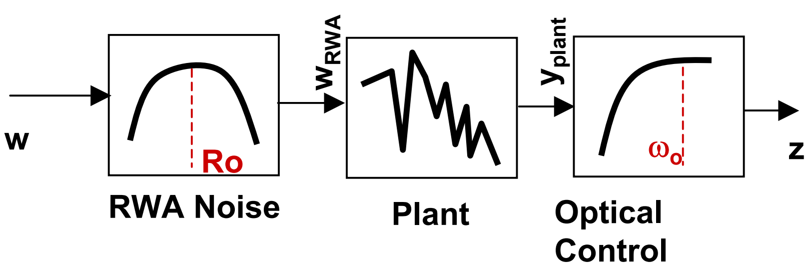
Isoperformance Analysis for SIM (2)

Taylor Exp: $$\sigma_z(p) = \sigma_z(p_k) + \underbrace{\nabla \sigma_z^T \Big|_{p_k} \Delta p}_{\text{first order term}} + \underbrace{\frac{1}{2} \Delta p^T \ H \Big|_{p_k} \Delta p}_{\text{second order term}} + \text{HOT}$$

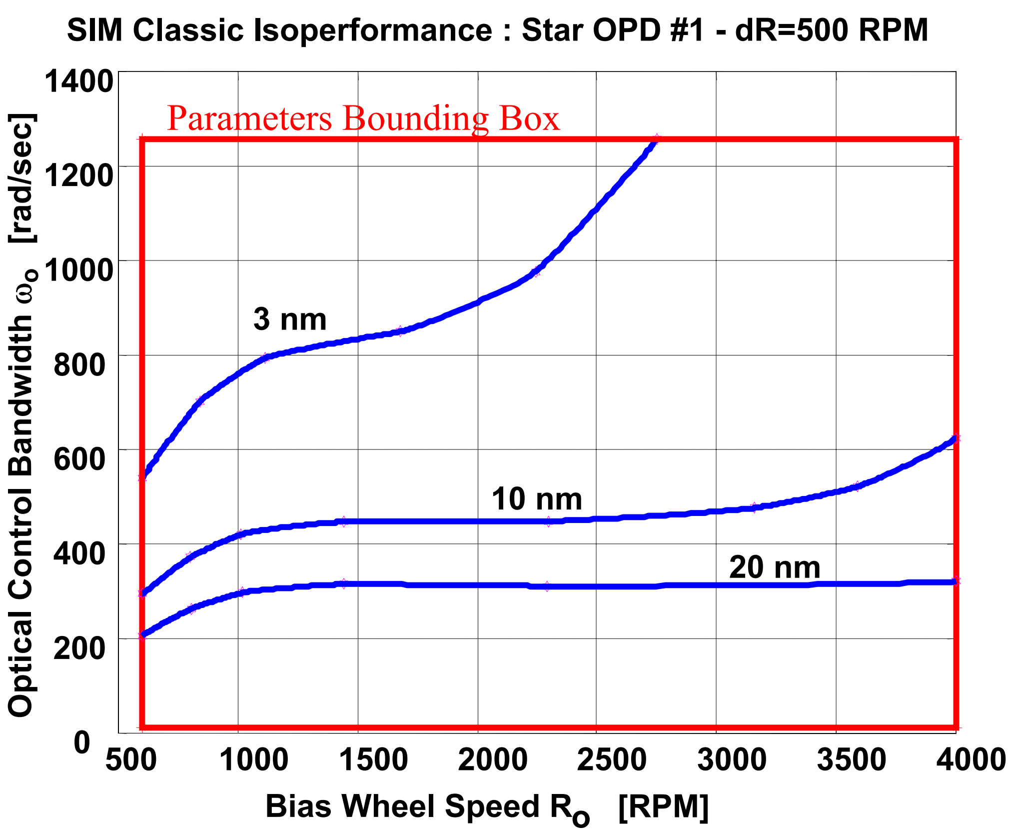
Then Solve: $$\nabla \sigma_z^T \Big|_{p_k} \Delta p \equiv 0$$

Version 2.0

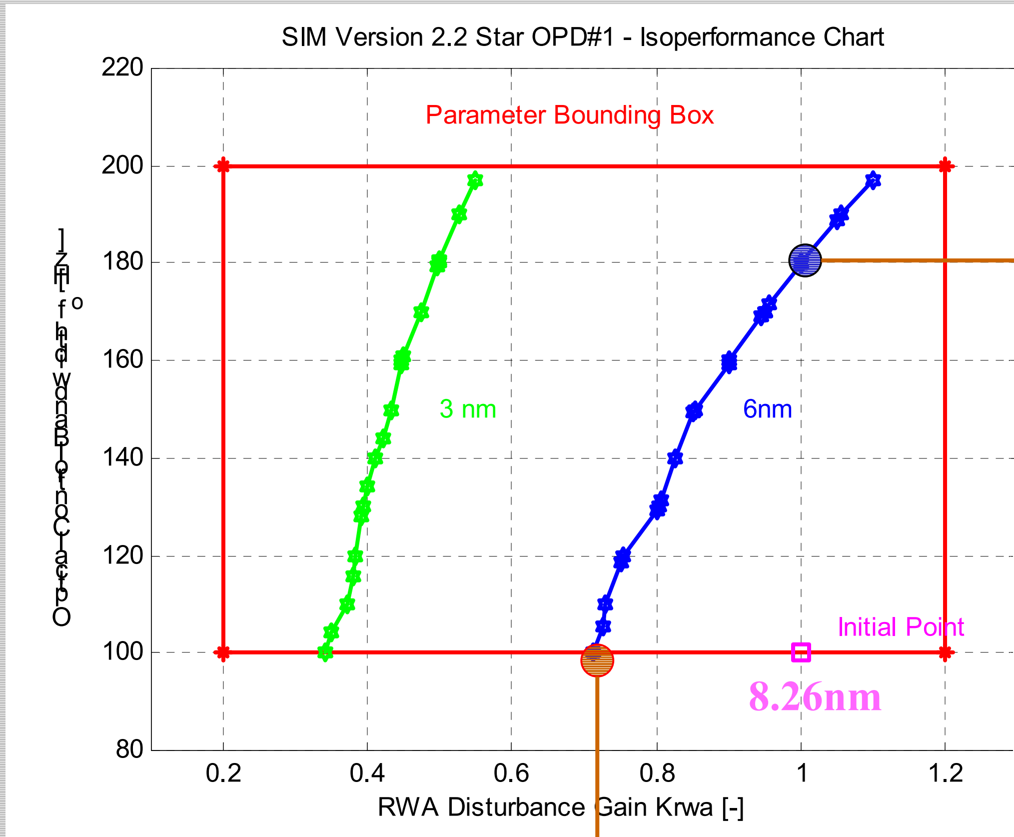
Treat RMS performances $\sigma_{z',i}$ of a dynamic opto-mechanical system as a constraint while trading key disturbance, plant and control parameters p_j with each other



Conclusion: As Bias Wheel Speed R_o increases control requirements become more stringent.



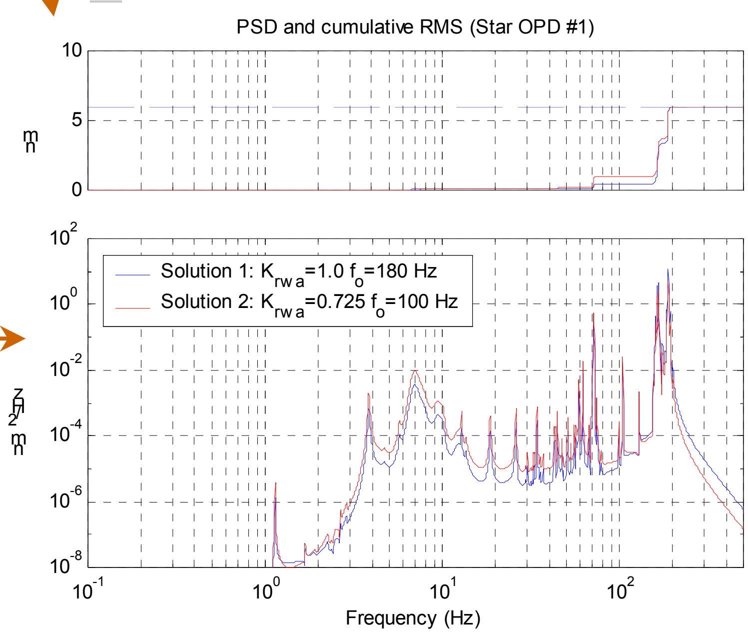
Isoperformance Results SIM V2.2



Results suggest that Star OPD #1 performance benefits more from reduction in wheel disturbance than from increasing optical control BW

Solution 2

Higher control bandwidth (180 Hz) and current wheels

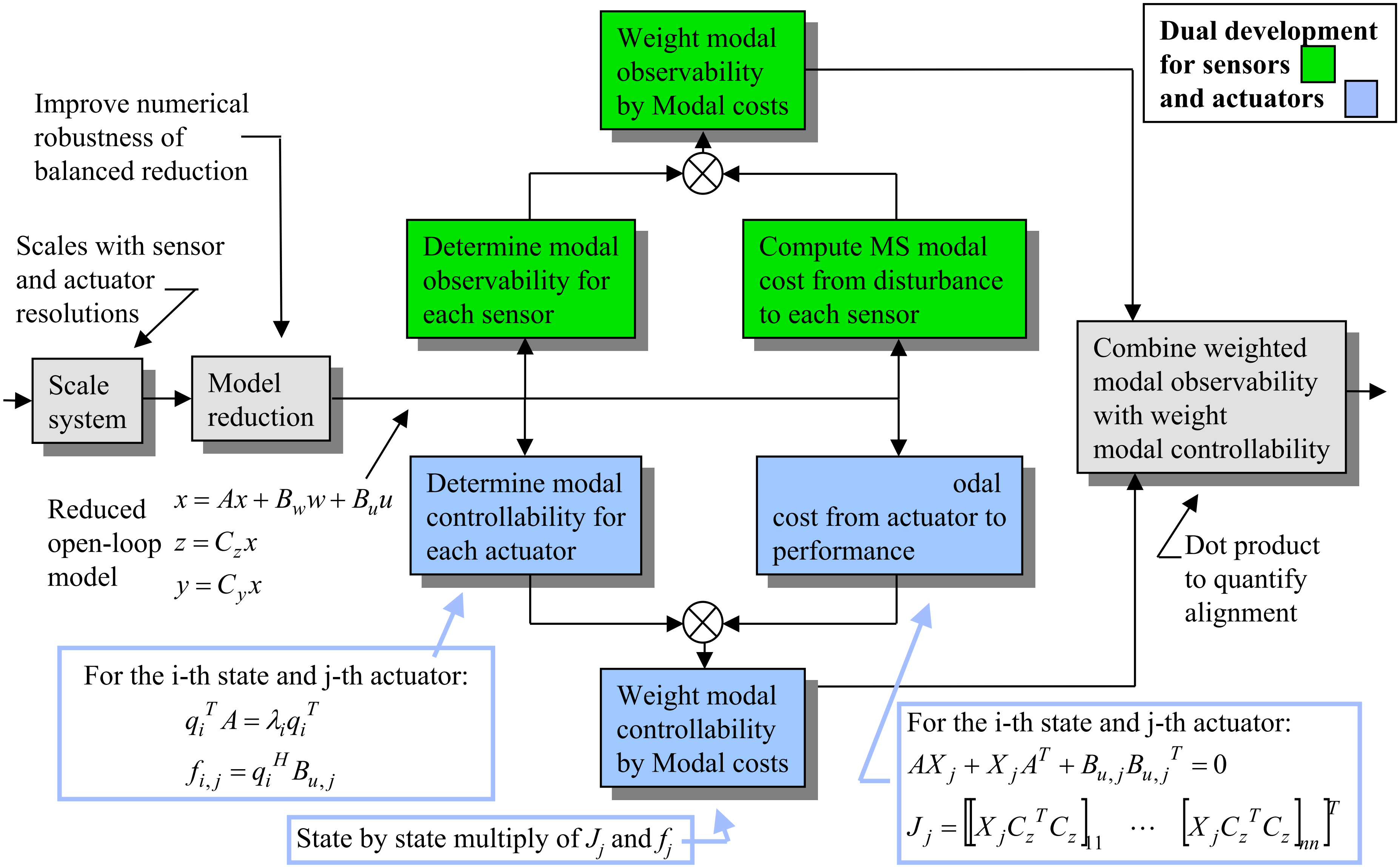


2% tolerance

Solution 1

Reduced wheel disturbance (0.75) and 100 Hz optical control

Model Sensor/Actuator Topology



SIM: Sensor/Actuator Effectiveness Matrix

- Two interferometers: guide 1 and guide 2
- Actuators:
  - Voice coil
  - Piezo (PZT) with mirror
  - Tip fast-steering mirror (FSM)
  - Tilt FSM
- Sensors:
  - Total differential pathlength (DPL)
  - Internal DPL
  - x-direction wavefront tilt
  - y-direction wavefront tilt

	Voice coil	PZT mirror	Voice coil	PZT mirror	Tip	Tilt	Tip	Tilt
Total DPL	3.8	6.7	0.9	3.2	4.2	-1.8	0.3	-3.1
Int. DPL	4.9	6.4	2.1	2.9	4.5	-1.4	0.4	-2.1
Total DPL	0.9	3.2	3.6	6.7	-0.1	-1.5	3.8	-0.2
Int. DPL	2.1	3.4	4.8	6.9	0.1	-1.7	4.4	0.5
Wavefront X tilt	-13.4	-3.6	-15.0	-7.1	6.3	-0.6	-0.3	-3.4
Wavefront Y tilt	-14.7	-23.3	-15.5	-22.7	0.5	3.1	-2.2	-3.6
Wavefront X tilt	-12.9	-7.5	-14.0	-4.0	0.5	-1.5	5.8	1.4
Wavefront Y tilt	-15.6	-23.9	-16.4	-23.4	-1.4	-2.9	1.2	3.2

Guide channel 1 (red) — Guide channel 1 (blue) — Phasing control block (green) — Fine-pointing control block (yellow) — Cross-coupling block (grey)

Control Tuning Framework

Formal Tuning Problem:

Minimize	Performance
subject to	(1) Stab. Robustness requirement
	(2) Limited deviation from baseline controller
	(3) Control channel gain limitations

Simplified Tuning Problem:

Minimize Augmented Cost:

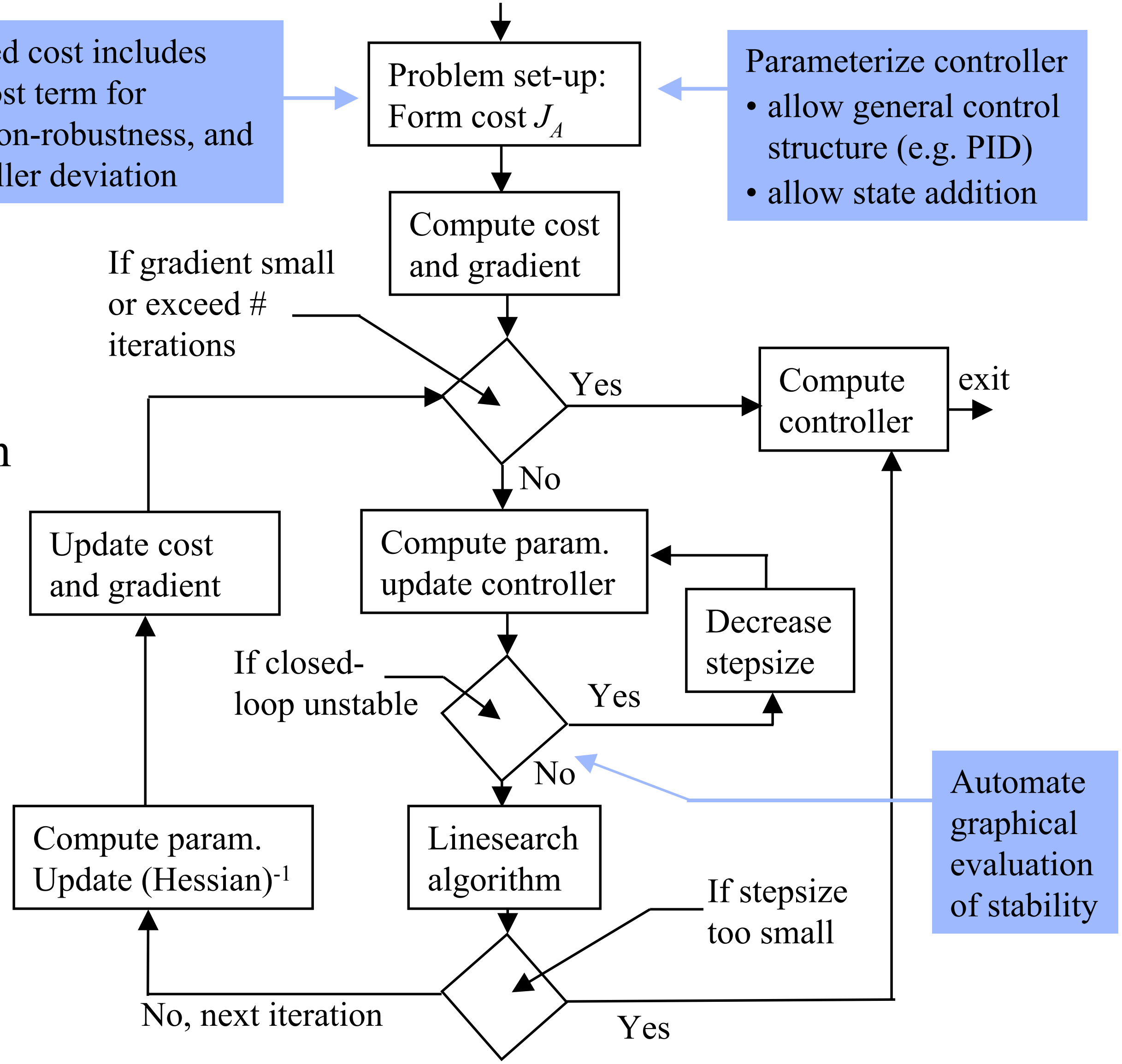
$J_A =$ Performance
+ Stab. Robustness metric
+ Penalty for baseline dev.
+ Penalty for control gain

- Gradients of each cost term are computable analytically
- Each cost term defined for (1) design model and (2) measured data

Control Tuning: Nonlinear Program

Cost expressions for either design model tuning or measured data tuning

Augmented cost includes explicit cost term for stability non-robustness, and for controller deviation



- Extension of MACE control design strategy
- BFGS nonlinear program
- Closed-loop stability-preserving iterations
- With each iteration a stabilizing tuned controller is designed

Thesis contribution

Controller Tuning: Design Knobs

Augmented Cost: $J_A(p) = J(p) + S_R(p) + d(p) + M(p)$

Performance

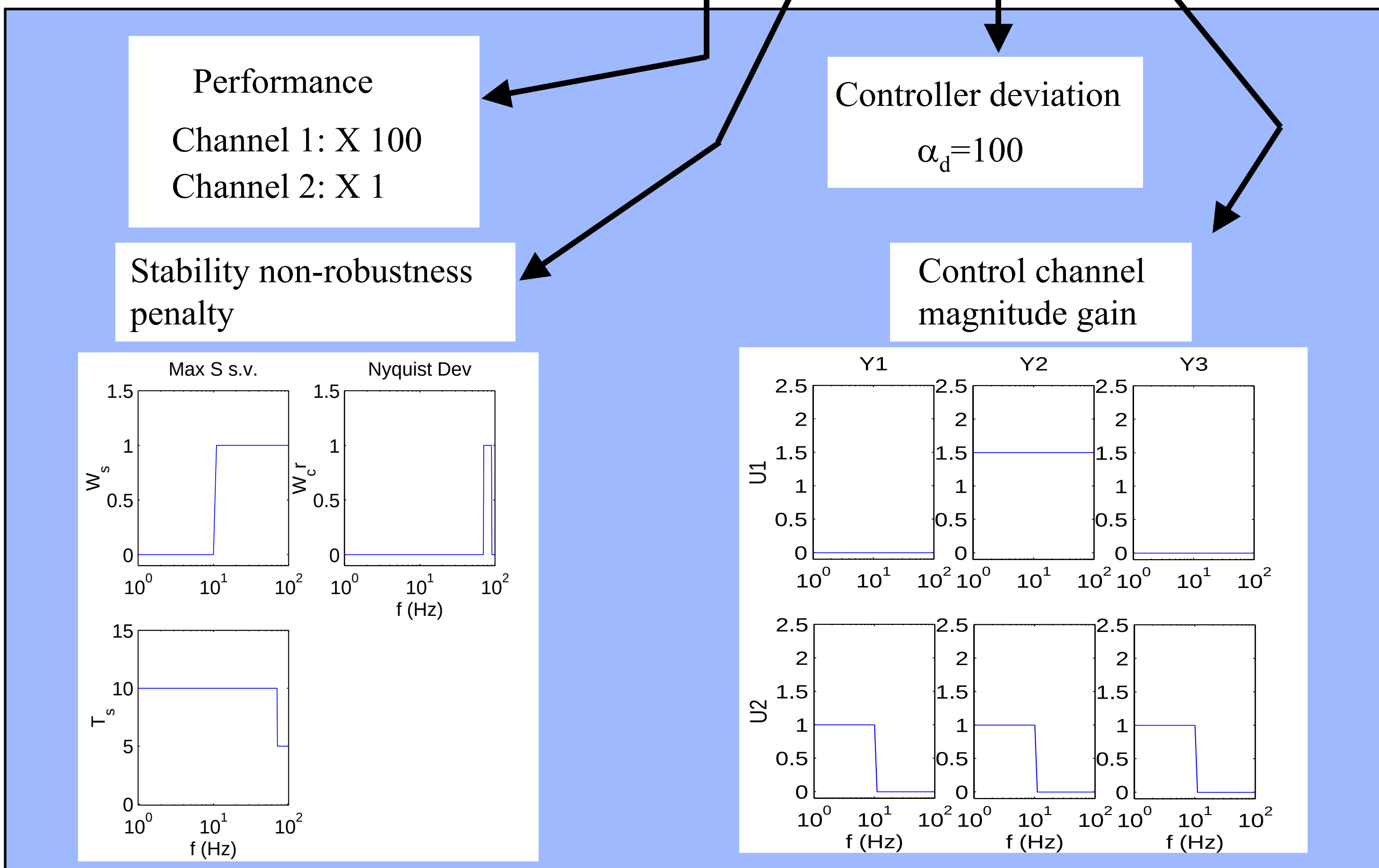
- Weight channel 1 to be more important than 2

Stability

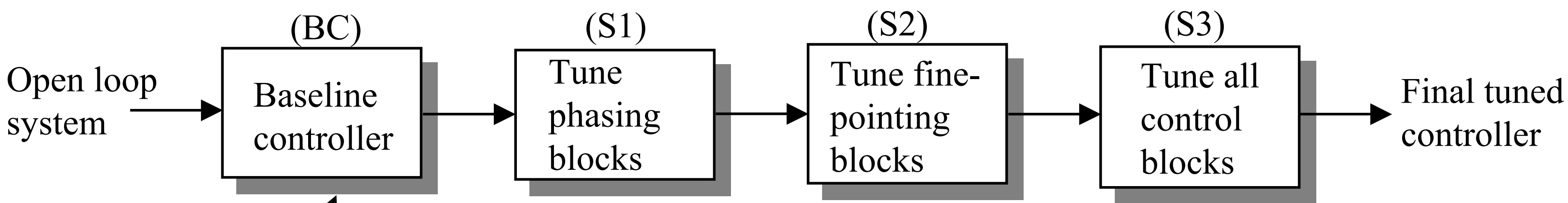
- Penalize Sens. s.v.>10dB bumps for f>10 Hz
- Roll-off with penalty on s.v.>5dB bumps for f>70 Hz
- Penalize a close pass of critical point for f~75 Hz

Channel Gain

- Penalize Y2 to U1
- Penalize low freq. use of U2



SIM Control Tuning: Family

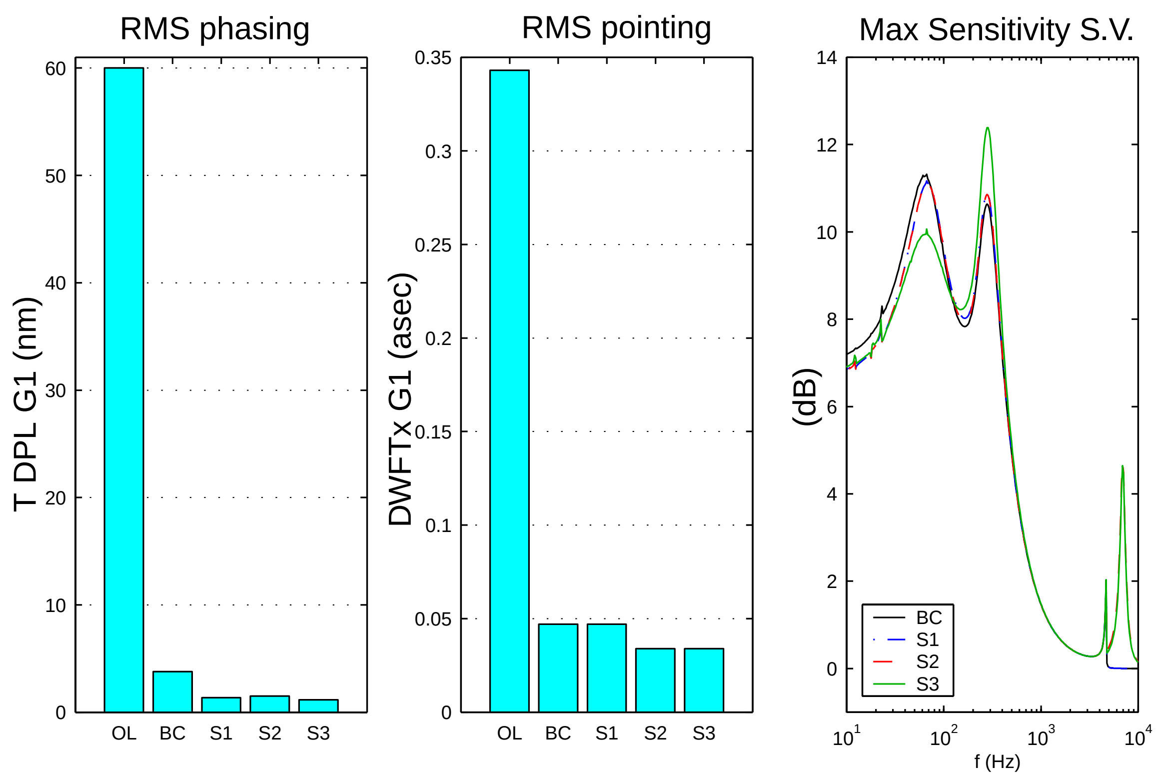


40 state JPL-designed
Guide 1 and 2 decoupled
phasing and pointing decoupled

- Tuning allows improvement in RMS phasing and pointing:

control	phasing (nm)	pointing (asec)
Baseline	3.8	0.0047
Tuned	1.2	0.0014

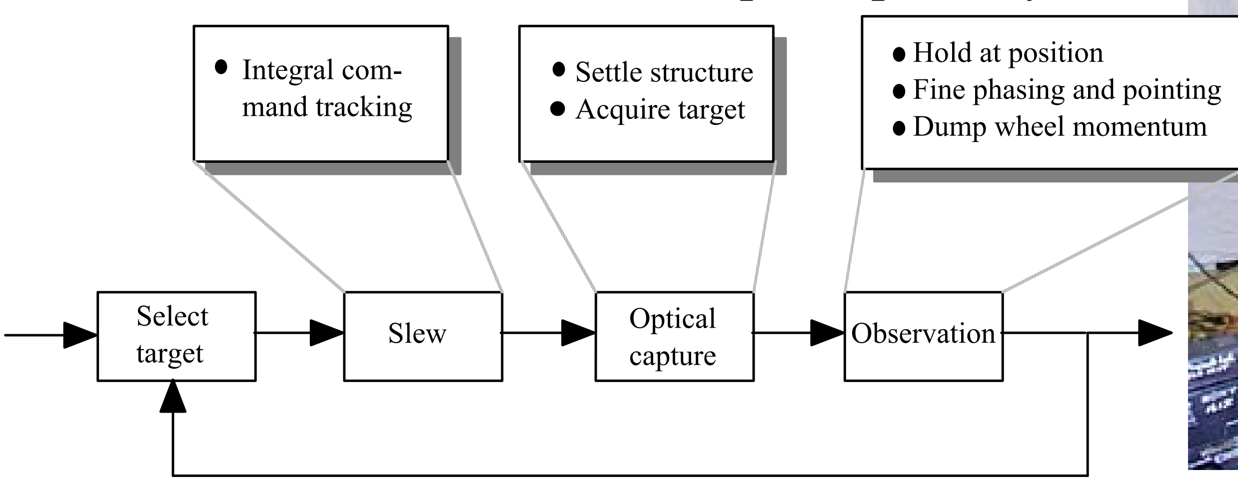
- Approach phasing nulling requirement
- Slight stability robustness penalty

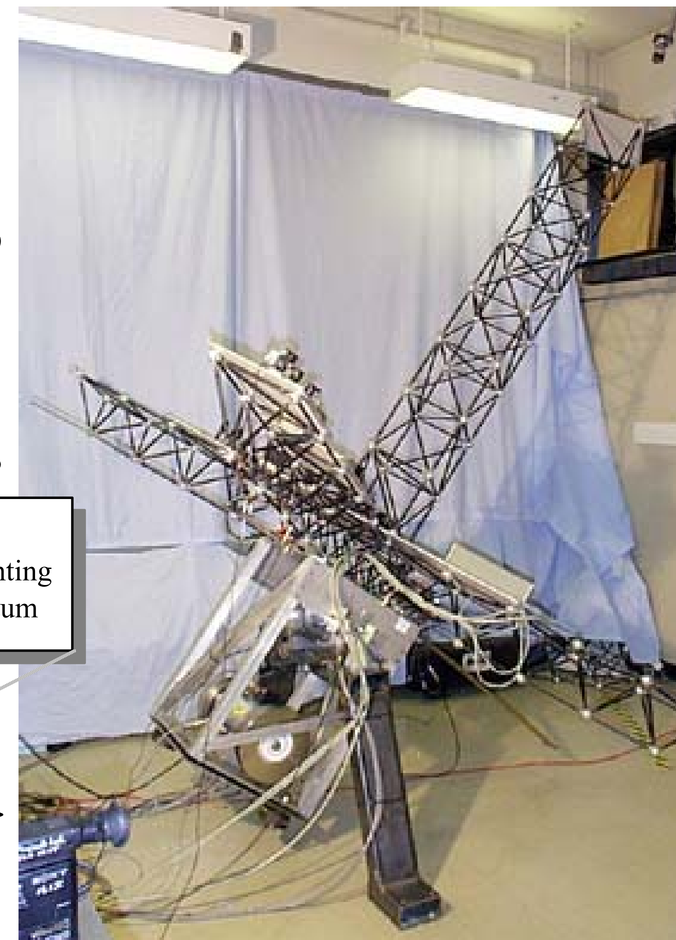


Origins Testbed

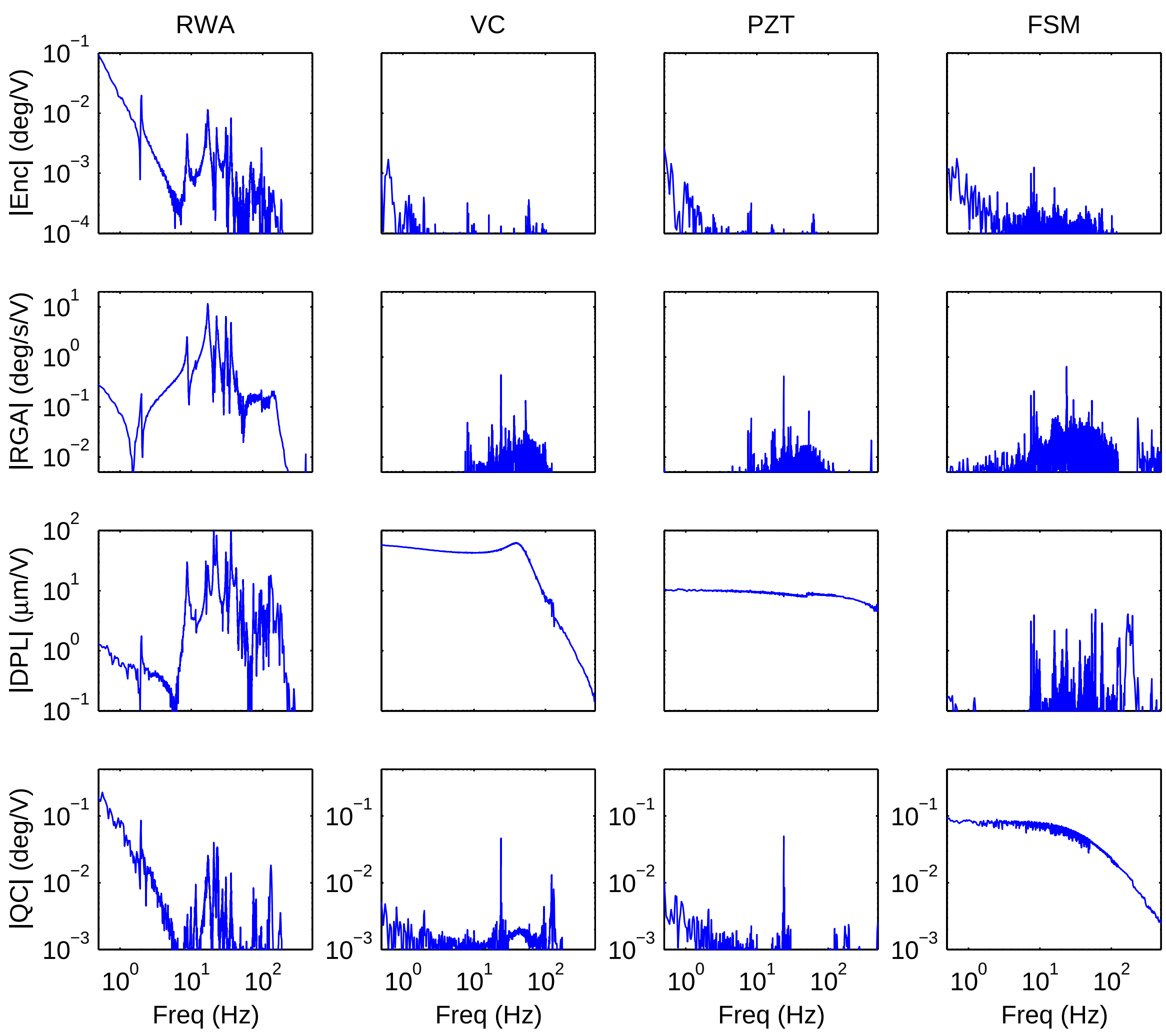
- Offers the combination of large angle slewing with fine phasing and pointing control in the presence of realistic disturbances
- Traceable to precision space telescopes, e.g. SIM, NGST

- Investigate methods for modeling and control of flexible structures
  - global MIMO control
  - vibration isolation and suppression schemes to meet future space-based telescope reqs.
  - dynamic system scaling issues
  - automation issues for complex optical systems





Origins Testbed: Transfer Function Matrix



Sensors:

- ENC - encoder
- RGA - rate-gyro assembly
- DPL - interferometer
- QC - quad cell photodiode

Actuators:

- RWA - reaction wheel
- VC - mirror on voice coil
- PZT - mirror on piezo stack
- FSM - fast steering mirror

Sensor / Actuator Index Matrix

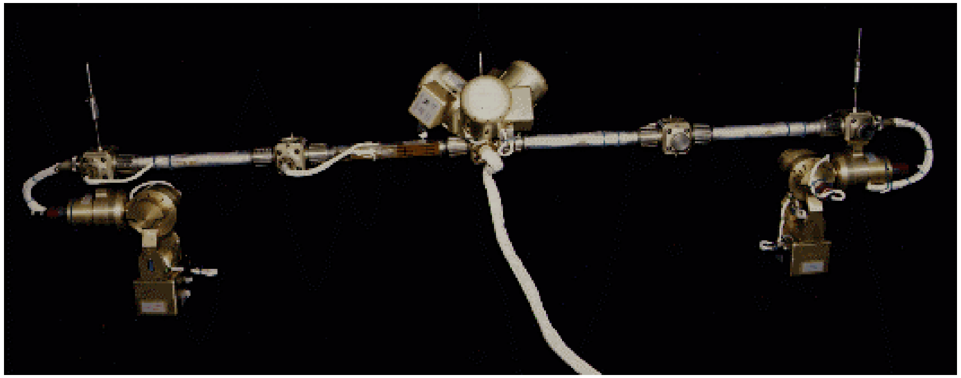
	RWA	**VC**	**PZT**	**FSM**
ENC	20.3	5.0	10.2	6.3
RGA	19.1	8.8	12.7	11.4
DPL	12.2	22.8	26.3	14.5
QC	17.4	8.9	13.7	20.6

MACE Flight Validation

Middeck Active Control Experiment (MACE)

1995: STS-67
2000:Currently an active experiment on ISS

Assessed effectiveness and predictability of advanced modeling and control algorithms on precision attitude and instrument pointing of a small satellite.



MACE Test Article

Demonstrated gravity effects can be accounted for during control design. For weakly nonlinear systems the accurate fit of measurement models can be deceptive.

Conclusions

- Despite differing scientific objectives, future space-based telescopes are dynamically similar.
- The MIT SSL has developed a framework for the Dynamics, Optics, Structures and Control (DOCS) for these telescopes.
- Flexible tools are developed and demonstrated in each of four critical areas
  - Modeling: physics-based FEM, model integration
  - Model Preparation: model reduction and conditioning, system ID
  - Analysis: disturbance, performance, sensitivity, and sensor/actuator coupling.
  - Design: isoperformance trades, control tuning
- Tools are validated on *large-order* models and on *experimental* test articles

Motivation

- Translate interferometer performance (null depth, sensitivity) to requirements on physical and optical motions
  - Aperture motion stability (AS)
  - Optical path difference (OPD)
  - Differential wavefront tilt (DWFT)
- Utilize the transmissivity function to characterize physical and optical effects on null depth

$$\Theta = \left| \sum_{k=1}^{N} D_k \exp\left(j2\pi \frac{L_k r}{\lambda} \cos\left(\bar{\theta}_k - \theta\right) \right) \exp\left(j\phi_k \right) \right|^2$$

$$\text{Null Depth} = \frac{\left\langle \Theta \right\rangle \big|_{\gamma_o}}{\left\langle \Theta \right\rangle \big|_{\max}}$$

- Derive control requirements from the perturbed transmissivity function

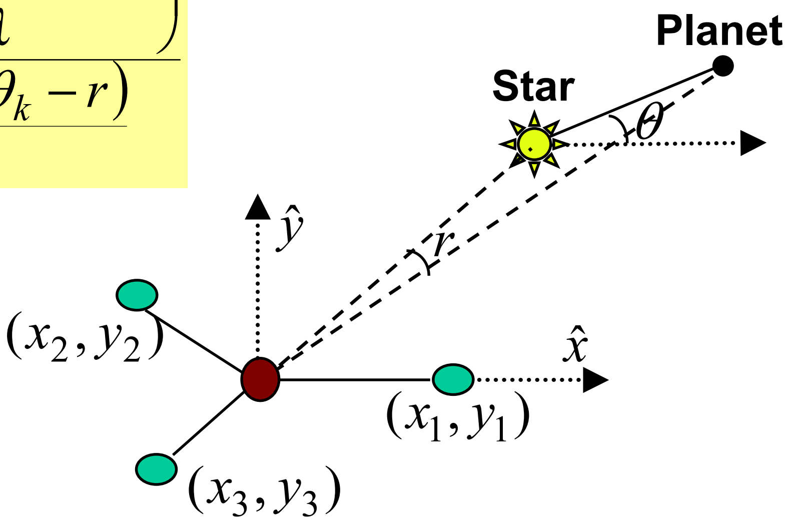
Development of Stability Requirements I

- Describe transmissivity function of a two-dimensional aperture array as

$$\Theta = \left| \sum_{k=1}^{N} G_k(\theta_k) \exp\left(j\left[\frac{2\pi r}{\lambda} x_k \cos(\theta) + \frac{2\pi r}{\lambda} y_k \sin(\theta) + \phi_k \right] \right) \right|^2$$

Circular Aperture

$$\mathrm{G_k}(\theta_k) = \frac{\pi D_k}{2\lambda}(1 + \cos(\theta_k - r)) \frac{J1\left(\frac{\pi D_k \sin(\theta_k - r)}{\lambda} \right)}{\frac{\pi D_k \sin(\theta_k - r)}{\lambda}}$$



D_k = diameter of k$^{\mathrm{th}}$ aperture

θ_{k} = tilt angle of k$^{\mathrm{th}}$ aperture

$(\mathrm{x_k}, y_k)$ = 2-D location of k$^{\mathrm{th}}$ aperture

ϕ_k = phase shift of k$^{\mathrm{th}}$ aperture

$(\gamma \cos(\theta), \gamma \sin(\theta))$ = image angular coordinates, $\gamma = \frac{2\pi \mathrm{r}}{\lambda}$

Development of Stability Requirements II

- Add small perturbations to transmissivity function

$$\Theta = \sum_{k=1}^{N} G_k^2 (\theta_k + \delta\theta_k) + \sum_{i=1}^{N-1} \sum_{l=i+1}^{N} G_i(\theta_i + \delta\theta_i) G_l(\theta_l + \delta\theta_l) \cos\left(\beta_{il} + \gamma \cos(\theta)\delta x_{il} + \gamma \cos(\theta)\delta y_{il} + \frac{2\pi}{\lambda}\delta P_{il} \right)$$

$$\beta_{il} = \gamma x_{il}\cos(\theta) + \gamma y_{il}\cos(\theta) + \phi_i - \phi_l$$

$$x_{il} = x_i - x_l, \qquad y_{il} = y_i - y_l$$

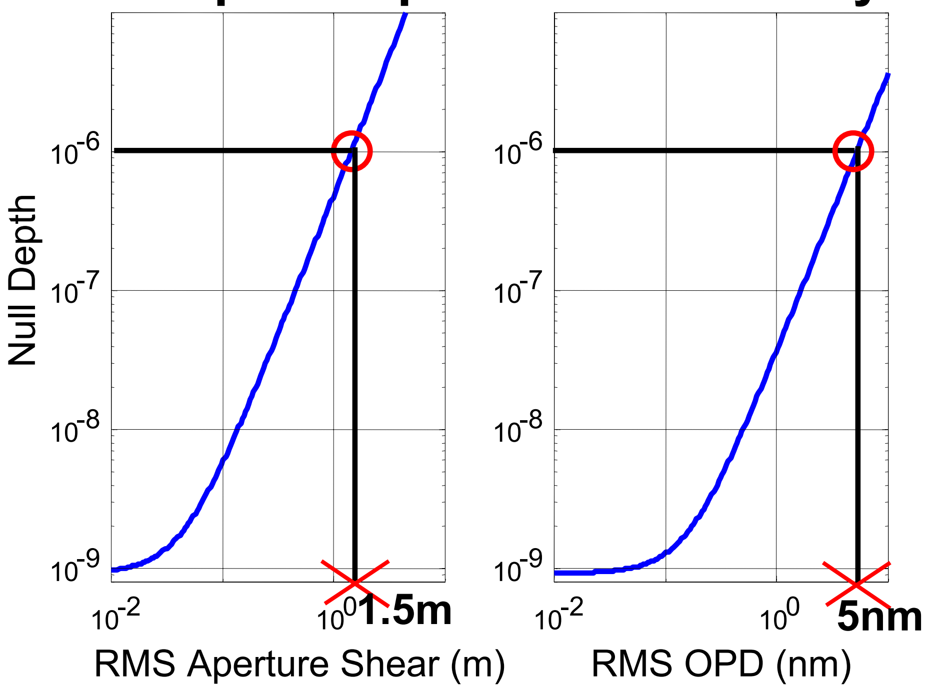
- Assume zero-mean, white Gaussian perturbations:

  $\delta\theta_k$ = aperture tilt angle

  $(\delta x_{il}, \delta y_{il})$ = aperture motion

  δP_{il} = differential pathlength

Example: 4 aperture linear array



Derive Performance Penalty Matrix

- Utilize the transmissivity function to generate performance requirements

$$\langle \Theta \rangle = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \Theta \, dt$$

$$\langle \Theta \rangle = \underbrace{\sum_{k=1}^{N} G_k^2(\theta_k) + 2\sum_{i=1}^{N-1}\sum_{l=i+1}^{N} G_i(\theta_i) G_l(\theta_l) \cos(\beta_{il})}_{\text{nominal}} +$$

$$\underbrace{\sum_{i=1}^{N-1}\sum_{l=i+1}^{N} -2G_i(\theta_i) G_l(\theta_l) \cos(\beta_{il}) \left[\frac{1}{2}\left(\frac{2\pi}{\lambda}\right)^2 \sigma^2_{P_{il}^2} + \frac{1}{2}\gamma^2 \sigma^2_{\Delta_{il}^2} + \gamma\left(\frac{2\pi}{\lambda}\right)\sigma^2_{\Delta_{il} P_{il}} \right]}_{\text{perturbation}}$$

- Define system cost from perturbation terms and write it in a matrix form

$$J = \lim_{t\to\infty} E\left[z^T Q_{zz} z \right] \qquad z = \left[P_{12}\ P_{13}\ \ldots\ P_{1N},\ \Delta_{12}\ \Delta_{13}\ \ldots\ \Delta_{1N} \right]^T$$

Q_{zz} = Cost matrix depending on the coefficients of the perturbation terms

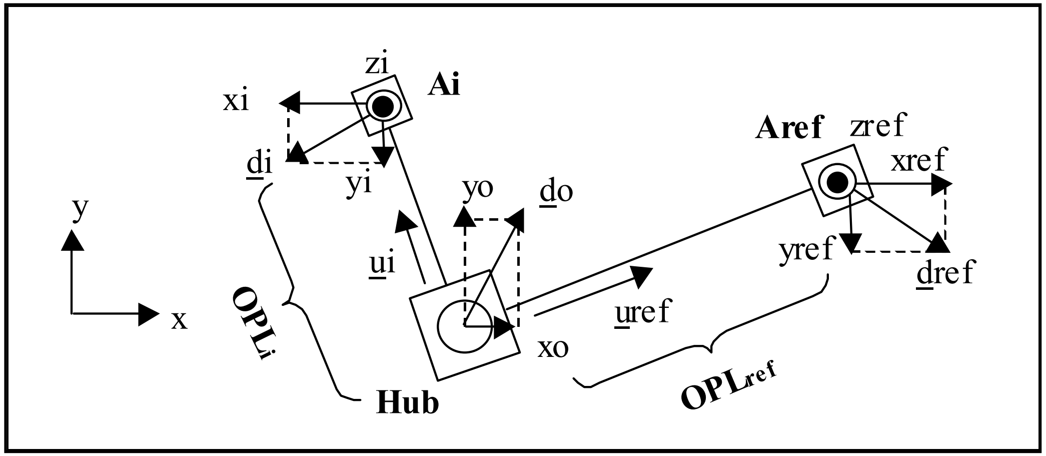
Optical Sensitivities

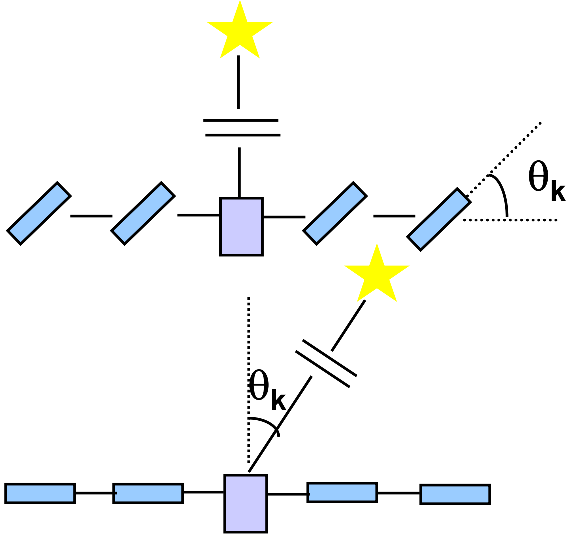
- Map physical coordinates to optical sensitivities
  - Optical Path Difference (OPD): $OPD_{ri} = OPL_{ref} - OPL_i$

$$OPD_{ri} = z_i - z_{ref} + (\bar{d}_{ref} \bullet \bar{u}_{ref} - \bar{d}_o \bullet \bar{u}_{ref}) - (\bar{d}_i \bullet \bar{u}_i - \bar{d}_o \bullet \bar{u}_i)$$

  - Aperture Shear (AS): $AS_{ri} = x_{ref} - x_i$
  - Aperture Tilt (AT): $AT_k = \delta\theta_{yk}$
- **Define transformation matrix $T_{z\zeta}$:** $z = T_{z\zeta}\zeta$

$z =$ performance vector $\quad \zeta =$ position and attitude of each s/c





* Figure courtesy of Olivier de Weck

Control Effort Analysis

- Assess control effort for different baseline configurations
  - Perform small perturbation analysis on the system to generate a set of linear equations which describe system dynamics

$$\dot{\zeta} = A\zeta + B_u u + B_w w$$

$$z = T_{z\zeta}\zeta$$

  - Solve the standard Linear Quadratic Regulator (LQR) problem

$$J = \int_0^\infty (\zeta^T T_{z\zeta}^T Q_{zz} T_{z\zeta}\zeta + u^T R_{uu} u)\,\mathrm{dt}$$

$$u = -K\zeta$$

  - Compute the closed-loop control covariance matrix

$$\Sigma_u = E\left[uu^T\right] = K\Sigma_{\zeta\zeta}K^T$$

$\Sigma_{\zeta\zeta} =$ **Closed-loop steady-state covariance matrix**

$$A_{cl}\Sigma_{\zeta\zeta} + \Sigma_{\zeta\zeta}A_{cl}^T + B_w B_w^T = 0$$