TPF Systems Analysis

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GINA: Systems Approach

- **Generalized Information Network Analysis (GINA) methodology**
  - *A systems engineering and architecting methodology, based on information network theory, that facilitates quantitative comparisons between viable architectures competing to satisfy a mission’s needs*

- **Comprehensive Metric Set**
  - ***Capability “Quality of Service” Metrics***
    - Isolation - ability to separate the desired signal from competing signals
    - Integrity - quality of signal characterized by noise or anomalies
    - Rate - throughput of the system
    - Availability - temporal and spatial variability of isolation, integrity & rate
  - ***Evaluation Metrics***
    - Performance - productivity over mission lifetime in presence of failures
    - Cost per Function - mission efficiency: lifecycle cost per performance
  - ***Adaptability** - **sensitivity analysis***

- **GINA derives these metrics from physics models**

GINA - TPF Metrics Capture

Rate

Integration Time

- *Zodiacal Distribution*
- *Collecting Area*
- *Detector Noise*
- *Propulsion Profile*

Availability

Operational Efficiency

- *Calibration*
- *Retargeting (slews)*
- *Deployment Time*
- *Anomaly Recovery*
- *Alignment*

Isolation

Transmissivity & EE

- *No. of Apertures*
- *Maximum Baseline*
- *Relative Geometry*

Integrity

Signal-to-Noise

- *Detector Noise*
- *Optical Bandpass*
- *Center Wavelength*
- *Mirror Surface Quality*
- *Aperture Diameter*
- *Thermal Noise*
- *Zodiacal Noise*
- *Glint*

Performance

Productivity

- *Mean time to failure*
- *Mission Lifetime*
- *Rate times Availability*

Cost

Lifecycle

- *P/L - aperture diameter*
- *Bus - mass & power*
- *Launch - mass & orbit*
- *Ops - complexity & orbit*
- *Learning Curve*
- *Thermal Shield Develop.*

TPF
Terrestrial
Planet Finder

GINA - Metrics Matrix

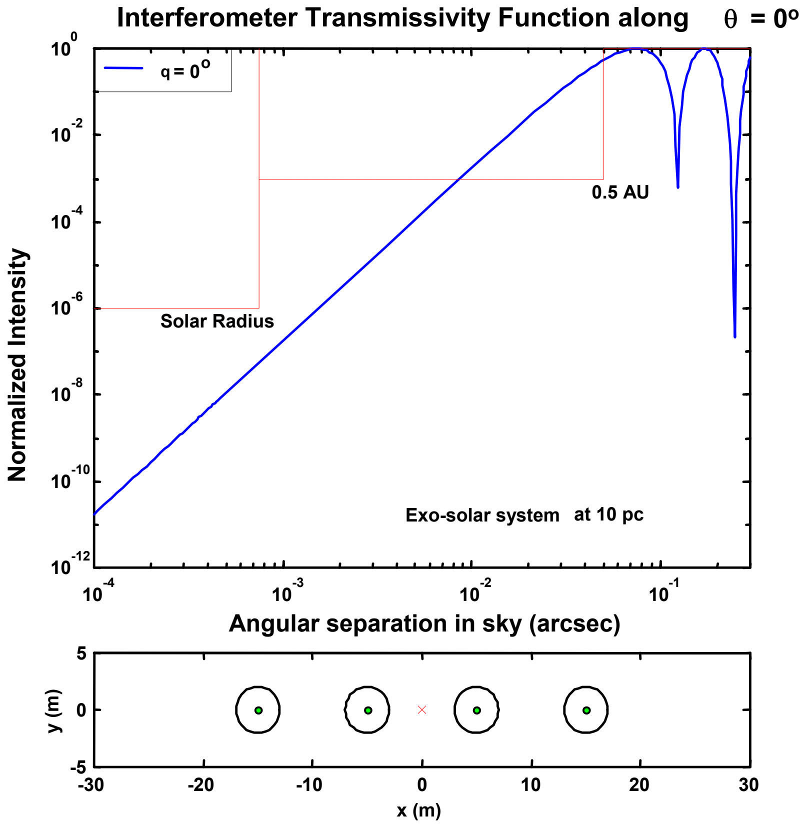
Metrics / **Trades**	*Isolation* *(Angular Resolution)*	*Rate* *(Images per unit time)*	*Integrity* *(SNR & **u-v coverage**)*	*Availability* *(Overhead time)*
Orbit **(Earth Trailing, Halo, L_2)**	**N/A**	🟩 **Different noise environments & comm. delays influence rate**	▨ **Different local zodical noise & solar thermal flux levels**	🟧 **Different comm. delays affect non-autonomous calibration**
Interferometer Type *(SCI,SSI,* **TSI***)*	🟩 **SSI & TSI have flexibility of changing baselines**	⬛ **SSI prop. & TSI power sensitive to rate changes. Different imaging modes**	⬜ **SCI - dynamics noise, SSI - alignment noise, TSI - dyn. & alignment**	🟧 **Different safe mode complexity and unique calibration events**
Number of Apertures *(4 to 10)*	🟦 **More apertures help tuning of interferometer transmission**	🟦 **Increased collecting area improves rate**	🟦 **Tuning of transmission suppresses starlight and exo-zodi dust**	🟩 **Operational complexity & graceful degradation**
Size of Apertures *(1 to 4 m in diameter;* **4 to 8 m in length***)*	**N/A**	🟦 **Increased collecting area improves rate**	🟩 **An increased size narrows FOV that will collect less local zodi**	**N/A**
Aperture Type *(Circular or* **Rectangular***)*	🟦 **Interferometer transmission sensitive to aspect ratio**	🟦 **Longer dimension of rectangle enables quick u-v coverage**	🟦 **Rectangle has more mirror area on axis, better SCI - u-v coverage**	**N/A**
Interferometer Baseline **(30 - 120 m)**	🟦 **Baseline drives angular resolution & transmission tuning**	**N/A** Only as Isolation & Integrity drive rate	🟦 **Tuning of transmission suppresses starlight and exo-zodi dust**	🟧 **Calibration complexity increases w/ distance between elements**
Wavelength **(7,10,20 μm)**	🟦 **Transmission tuning less effective at longer wavelengths**	**N/A** Detections & spectroscopic images span 7-17 μm	🟦 **Longer wavelengths diminish null & increase instrument noise**	**N/A**

Legend
🟦 **Aperture Physics**
⬜ **DOCS**
▨ **Environment**
🟩 **GINA**
🟧 **Operations**
⬛ **Spacecraft Bus**

Planet Detection Modeling

Transmissivity Function (Circular Apertures):

$$\Theta(\theta,r)=\frac{\pi^2(1+\cos\theta)^2}{\lambda^2}\left|\sum_{k=1}^{n} D_k \frac{J_1\left(\frac{\pi D_k\sin\theta}{\lambda}\right)}{\frac{\pi D_k\sin\theta}{\lambda}}\exp\left[j2\pi(L_k\,\theta/\lambda)\cos(\delta_k-\phi)\right]\exp(j\,\phi_k)\right|^2$$



n - number of apertures

D_k - aperture size

L_k - aperture length

θ - point source angular separation from star

λ - wavelength

d_k - aperture clock angle

ϕ - point source separation from interferometer

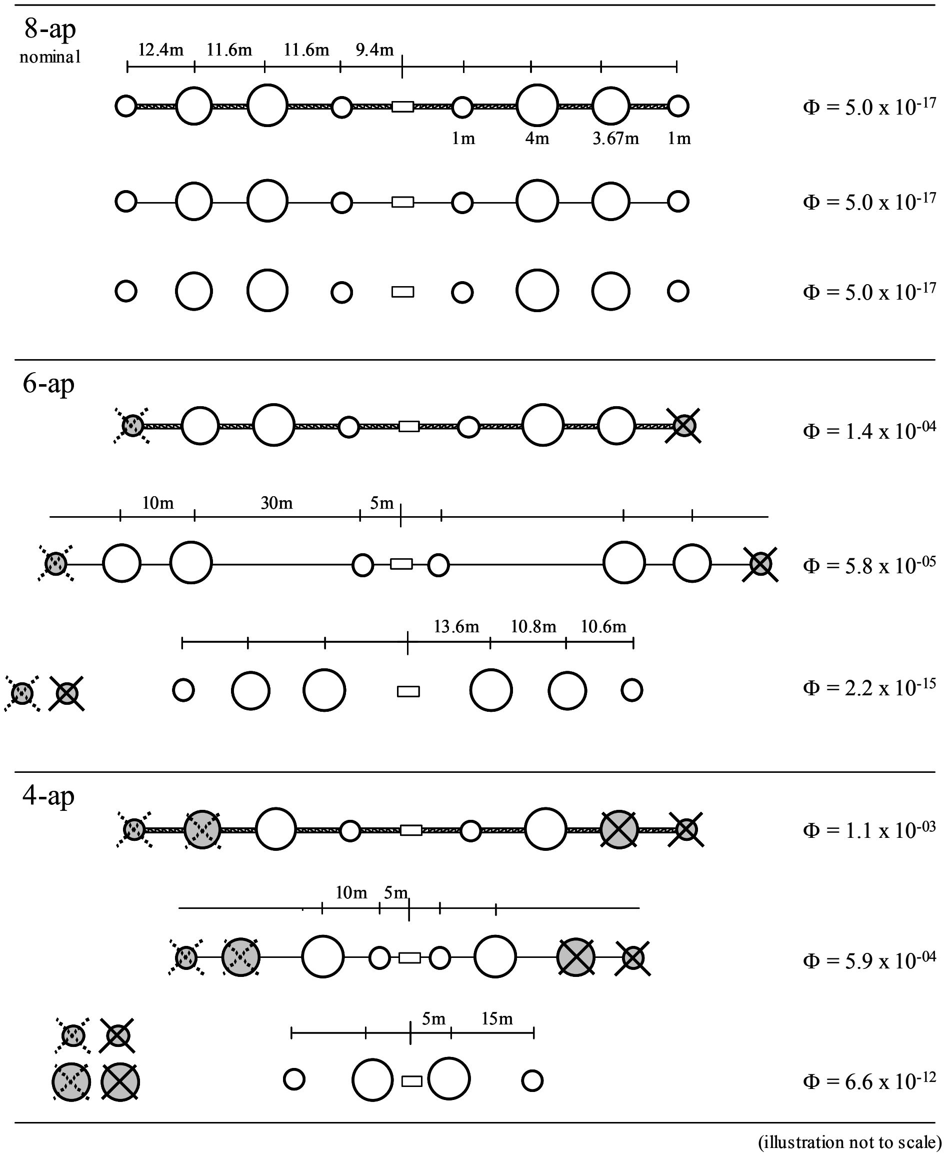
ϕ_k - independent phase shift for each aperture

Modeling Notes:

Nulling interferometer based upon Bracewell nulling interferometer (180° phase shift)

Optimization is performed a priori using simulated annealing technique or Brute force search algorithm

Graceful Degradation



(illustration not to scale)

- ***Graceful Degradation***

  Architectures that can reconfigured without a substantial loss in performance are said to degrade gracefully

  -SCIs have fixed aperture position, severely limiting performance as components fail

  -TSI apertures can be reconfigured in a line, improving performance in partially failed states

  -SSI apertures can maneuvered freely, giving it an even better performance during degradation

- ***Sub-optimization***

  If each collector pair has a unique size, there are (n/2)!/2! unique partially failed states. Each state corresponds to a potential aperture re-configuration, a sub-optimization, which differs depending on the interferometer type.

Astrophysical Imaging Modeling

Total Integration Time per Image

Derived from “Signal and Noise in Dilute Aperture Interferometric Imaging for TPF” by N. Woolf and is independent of interferometer type and imaging method.

Assume SNR = 100 and Spectral Resolution, R = 100.

$$SNR = \frac{SAt_{uv}}{\sqrt{(S+B_n)At_{uv} + D_n t_{uv} + R_n}}$$

solve for t_i

$$t_i = \frac{SNR^2 ND[D_n + A(B_n+S)]}{2A^3S^2}\left(1+\sqrt{\frac{4A^2S^2R_n}{SNR^2[D_n+A(B_n+S)]^2}+1}\right)$$

where:

$A \equiv$ total architecture area (m^2)

$B_n \equiv$ background radiation noise (counts/m^2/sec)

$D \equiv$ desired area to be simulated (m^2)

$D_n \equiv$ detector dark noise (counts/sec)

$N \equiv$ number of apertures (#)

$R_n \equiv$ detector read noise (counts)

$S \equiv$ source signal (counts/m^2/sec)

$t_i \equiv$ integration time per image (sec)

$t_{uv} \equiv$ integration time per uv point (sec) $= \dfrac{t_i A}{DN}$

Assuming u-v plane can be filled completely without duplication

where the design vector sets:

$A \equiv$ total architecture area (m^2)

$Bl \equiv$ interferometer baseline (m)

$D \equiv$ desired area to be simulated (m^2) $= \pi Bl^2/4$

$N \equiv$ number of apertures (#)

where the signal & noise terms can be approximated as:

$B_n \approx 15$ counts/m^2/sec

$D_n \approx 5$ counts/sec

$m \equiv$ interferometer efficiency $= 0.1$

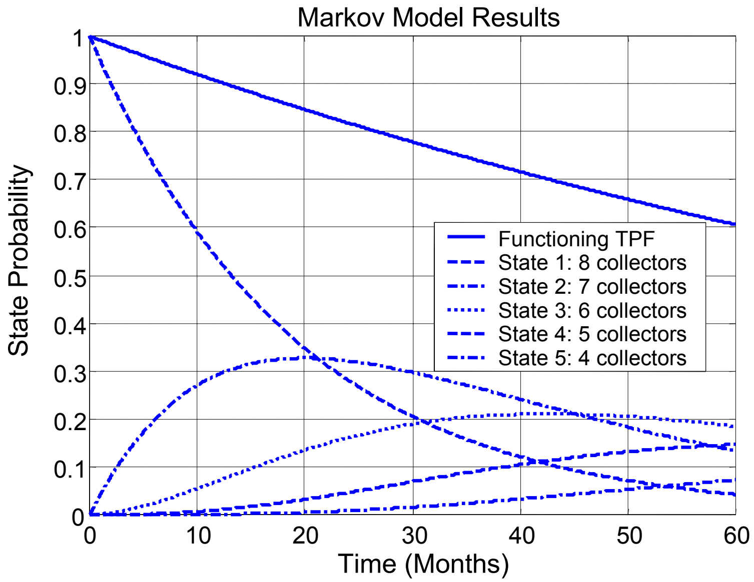
$R_n \approx 10$ counts

$S \approx 1500m = 150$ counts/m^2/sec

$SNR \equiv$ Signal to Noise Ratio $= 100$

Performance - Modeling

Markov Model used to calculate Steady State Probabilities (P_i)



Expected Utility (Total Number of Planetary Observations)

$$E(T)=\int_{74}^{256}\sum_{i=1}^{n}C_{si}P_i(t)dt+\int_{257}^{313}\sum_{i=1}^{n}C_{mi}P_i(t)dt+\int_{314}^{365}\sum_{i=1}^{n}C_{di}P_i(t)dt$$

$$+\int_{366}^{547}\sum_{i=1}^{n}C_{si}P_i(t)dt+\int_{548}^{658}\sum_{i=1}^{n}C_{mi}P_i(t)dt+\int_{659}^{730}\sum_{i=1}^{n}C_{di}P_i(t)dt$$

$$+\int_{731}^{804}\sum_{i=1}^{n}C_{si}P_i(t)dt+\int_{805}^{1004}\sum_{i=1}^{n}C_{mi}P_i(t)dt+\int_{1005}^{1095}\sum_{i=1}^{n}C_{di}P_i(t)dt$$

$$+\int_{1096}^{1132}\sum_{i=1}^{n}C_{si}P_i(t)dt+\int_{1133}^{1333}\sum_{i=1}^{n}C_{mi}P_i(t)dt+\int_{1334}^{1460}\sum_{i=1}^{n}C_{di}P_i(t)dt$$

$$+\int_{1461}^{1498}\sum_{i=1}^{n}C_{si}P_i(t)dt+\int_{1499}^{1680}\sum_{i=1}^{n}C_{mi}P_i(t)dt+\int_{1681}^{1825}\sum_{i=1}^{n}C_{di}P_i(t)dt$$

Each row represents a year of the mission life

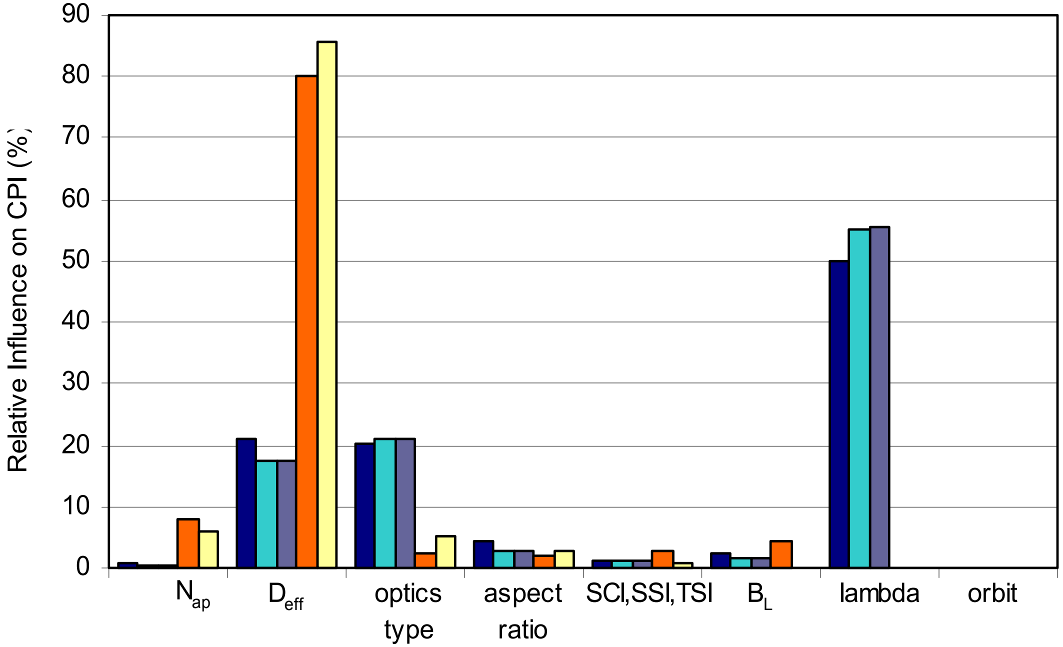
Each column represents a specific mission stage (survey, medium and deep spectroscopy).

Imaging Capability

$$C_{i\,j}=\left(\tau_{ij}+T_{ovhead_{ij}}+T_{recovery_{ij}}\right)^{-1}$$

$$\tau=\left[\frac{SNR\sqrt{\left(Q_{leak}+Q_{LZ}+Q_{EZ}+Q_{dark}+Q_{planet}+Q_{flat}\right)}}{Q_{planet}}\right]^2$$

Performance - ANOVA



Less Influential Variables

- **Interferometer Baseline**

  Baseline drives local performance & cost effects. For a common set of variables, baseline changes can always improve CPI (but only by a relatively small amount)

- **Orbit (Halo, Earth Trailing, and L_2)**

  For the chosen orbits, there are only negligible changes in local-zodi & detection performance.

More Influential Variables

- **Architecture Area**

  Drives transmissivity function and imaging model. Most influential design variable.

- **Aperture Type**

  Circular optics out-perform strips in detection operations. Modest benefit in imaging (competing factors).

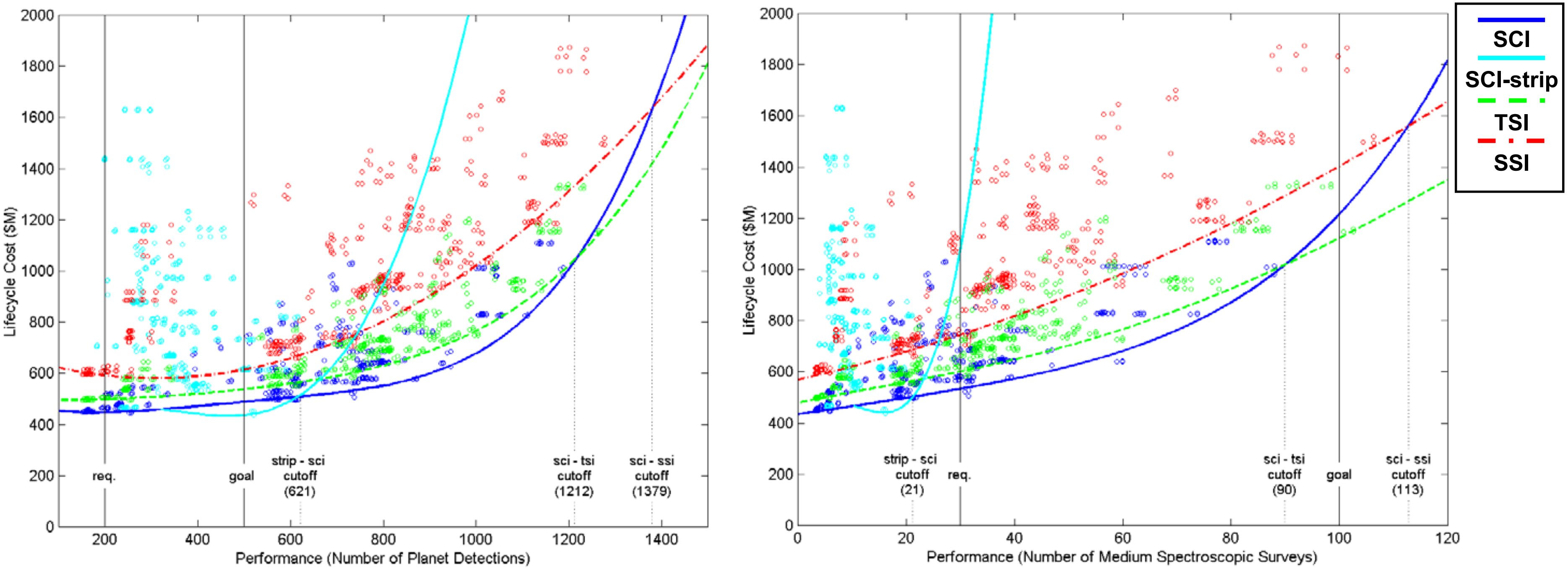
- **Interferometer Type**

  Architecture a clear cost driver. Graceful degradation effect on performance – a function of failure rate.

- **Wavelength**

  Most influential in detection ops, yet observations must span λ range – unless the mission requirements are compromised.

Planet Detection - SSI,TSI,SCI



Influence of Interferometer type

- *Interferometer type is influential in cost but not in performance. Hence the vertical (and not horizontal) separation of the pareto fronts.*

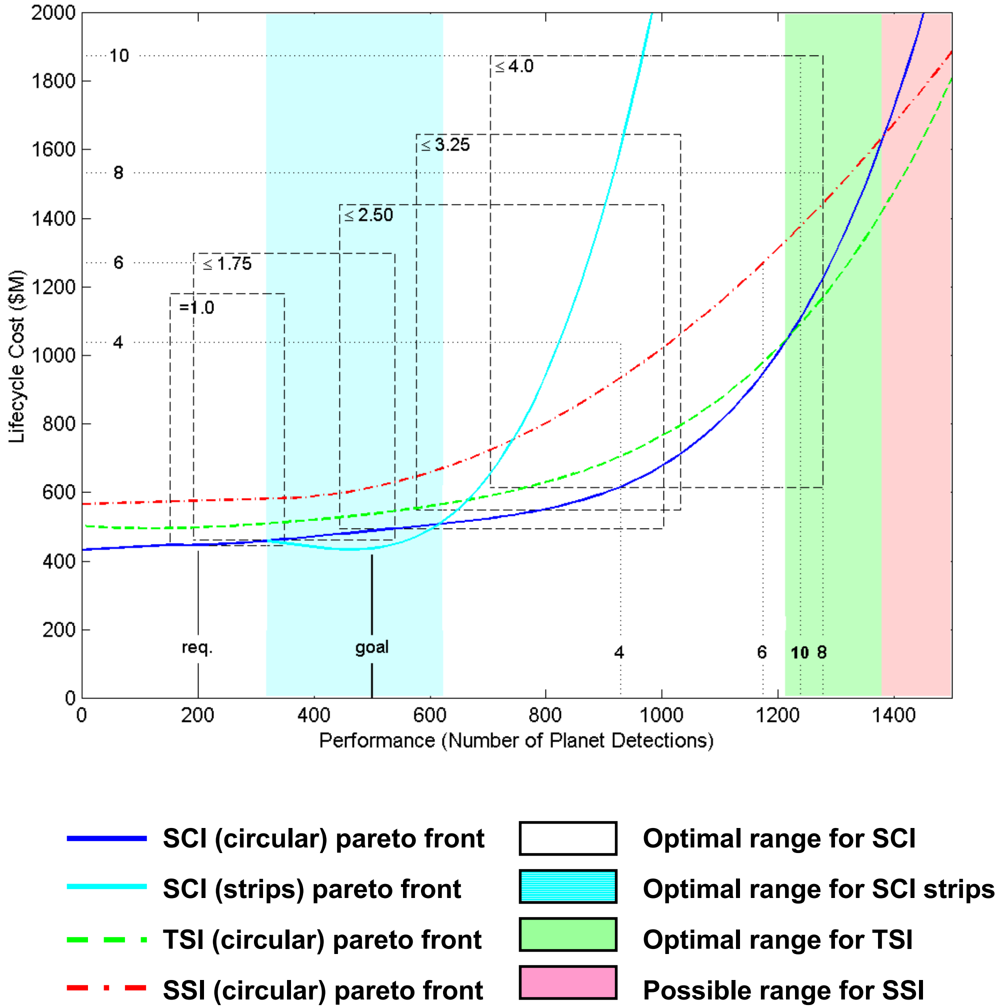
Interferometer type Evaluation

- ***SCI** is preferred for detection operations. In some cases, SCI-strips offer a cost benefit because its assumed that two strips are cut from a single circle aperture.*

Graceful Degradation of SCI, TSI, & SSI

- *Nominally, each interferometer type will have the same performance. It is in the presence of failures that it influences performance*
- *SCIs are fixed, severely limiting performance as soon as one collector fails.*
- *TSIs can be reconfigured in a line, improving performance in partially failed states.*
- *SSIs have an additional degree of freedom (dof) giving it an even better performance during degradation.*

Summary - Planet Detection



- **Effective Diameter**

  *Larger diameters enable better performance for modest cost increase (bottom edge of the rectangles). The planet detection requirement can be met with **D = 1m** and the goal can be met with **D = 1.75 m**.*

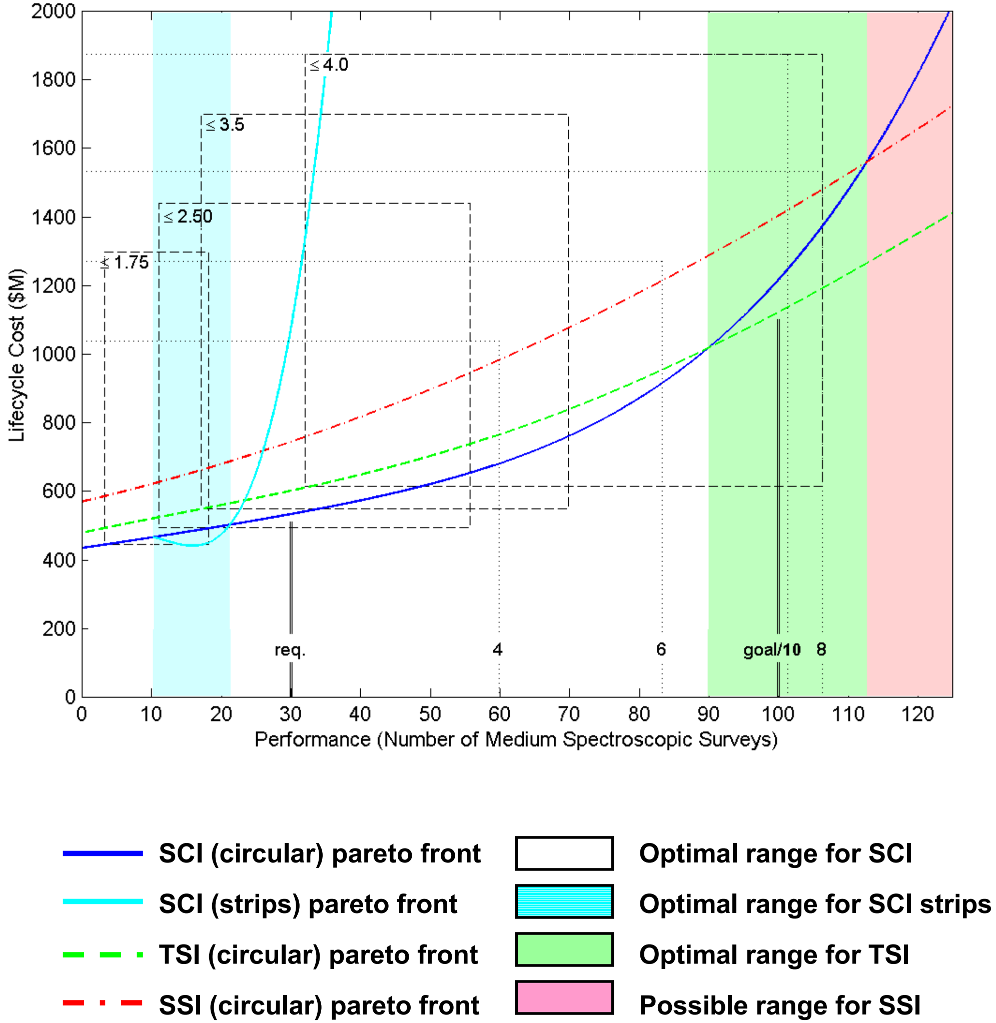
- **Number of Apertures**

  More apertures generally increase performance - until operational overhead and baseline tuning effects dominate (>8 aps). The planet detection requirement can be met with $N_{ap} = 4$.

- **Interferometer Type**

  *With the current operations timeline, **structurally connected interferometers** are adequate. Some strip architectures offer cost benefits, assuming multiple strips are cut from a circular aperture. Strips are cost prohibitive if **L >> 4**.*

Summary - Medium Spect.



- **Effective Diameter**

  The medium spectroscopy requirement can be met with ***D = 2.5 m*** *and the goal can be met with* ***D = 4 m****.*

- **Number of Apertures**

  The medium spectroscopy req. can be met with ***N_{ap} = 4****, but* ***8*** *or* ***10*** *are needed to meet the goal.*

- **Interferometer Type**

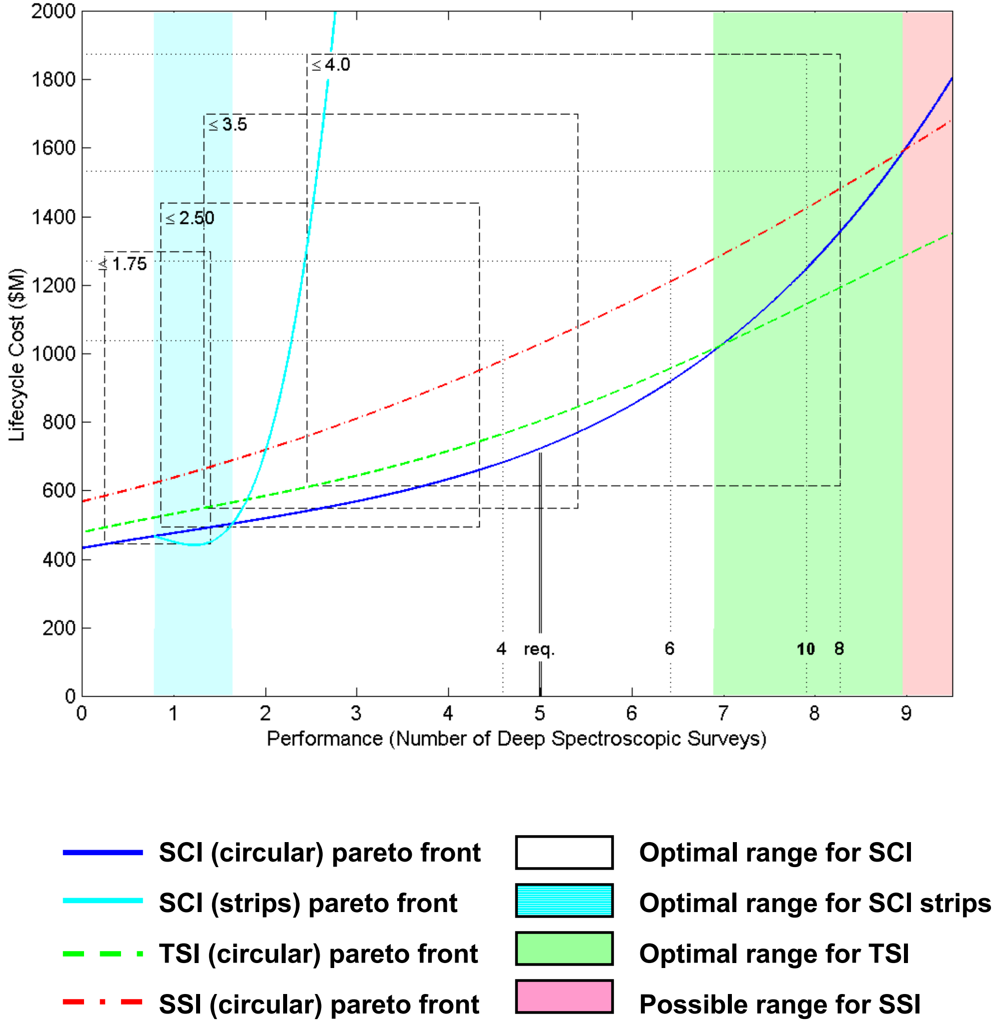
  The medium spectroscopy req. is best met w/ ***SCI****, but* ***TSI*** *is preferred to meet the goal.*

- **SNR and Timeline**

  SNR drives the pareto shape, But the timeline determines the performance value.

  [See Deep Spectroscopy Results]

Summary - Deep Spectroscopy



- **Effective Diameter**

  The deep spectroscopy requirement can be met with ***D = 3.5 m*** *– expectedly more stringent than med. spectroscopy.*

- **Number of Apertures**

  The deep spectroscopy req. can be met with $\boldsymbol{N_{ap} = 6}$*.*

- **Interferometer Type**

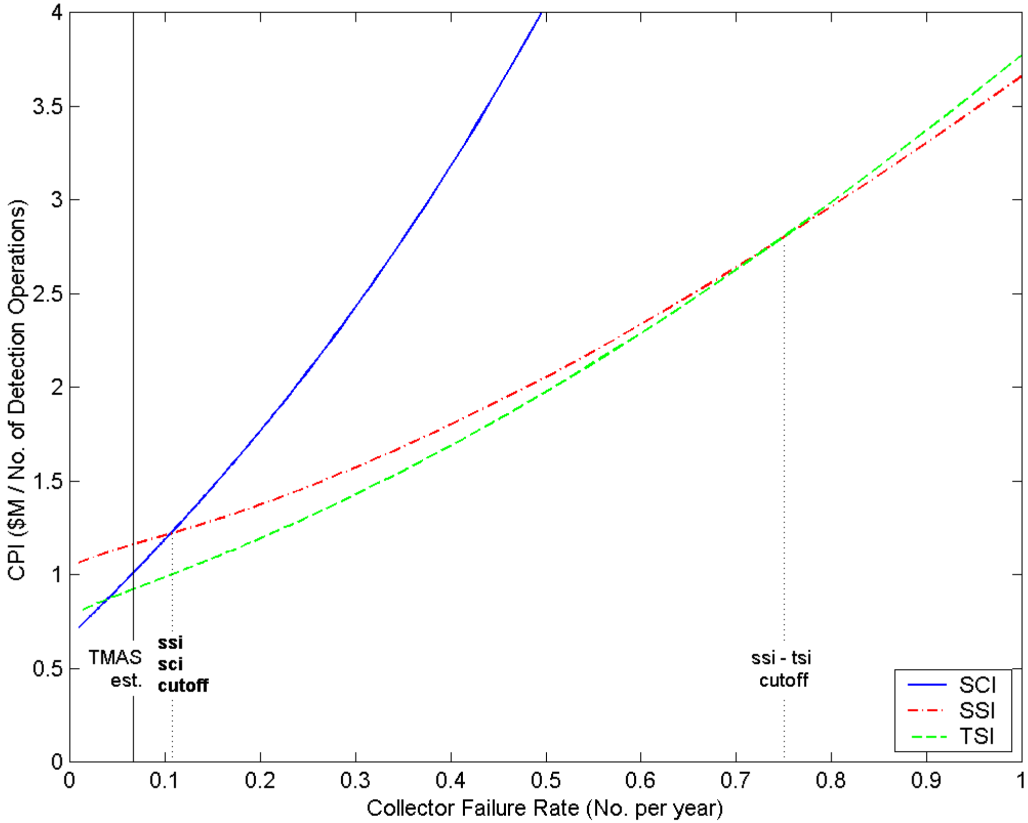
  The deep spectroscopy req. is best met w/ a ***structurally connected interferometer.***

- **SNR and Timeline**

  The deep spectroscopy results are similar in shape to the medium results – except with respect to their relative performance.

  This is likely caused by the disproportionate time allotted to each operational mode. Hence there should be an interesting time allocation optimization.

Regimes Favoring SSI



This plot shows the adaptability of CPI to changes in the collector failure rate for a specific architecture: 8-apertures, 110m baseline, variable-sized optics. Care should be taken when extrapolating this trend to other architectures in other regions of the trade space.

- **Long Baselines**

  As baseline increases, truss and/or tether mass becomes prohibitive.

  Longer baselines also increase collector centripetal acceleration and propellant - but a lower rate.

- **Slow Rotation Rates**

  As rotation rate decreases, centripetal acceleration for SSIs decreases. This drives down propellant and cost.

- **Risk prone Missions**

  As failure rate increases, SSI shows its graceful degradation capability, especially for arrays with variable-sized optics

  SSI architectures are preferred over SCI when failure rate is greater than 1/10 year $^{-1}$

Conclusions

- A 4-element structurally connected interferometer with D>2m **could meet a majority of the requirements** – planet detection, medium spectroscopy and short baseline imaging.
- The remaining requirements – deep spectroscopy and long baseline imaging may be met **if the operations timeline is optimized.**
- Tethered spacecraft interferometers appear to be a viable compromise between SCI and SSI although the dynamics and control complexity has not been modeled.
- While the technology for SSI is tractable, the aggressive maneuver (rotation) severely limits science throughput limiting SSI to longer baseline science. Key technologies are
  - *micro-spacecraft technology*
  - *electromagnetic formation flight*
  - *propellant replenishment*
  - *autonomous fleet control*