



A different approach to calculate the far-side seismic migration Green's functions



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Seismic images of magnetic activity on the far surface of the Sun are calculated using the seismic holography technique. The method is based on the comparison of waves going in and out a particular point (focus) in the Sun and needs of the Green's functions representing the disturbance measured in the front side (pupil) from a unit acoustic impulse originated at the focus. In this work we describe how to compute Green's functions for the Lagrangian pressure fluctuation from a spherical polar expansion of the adiabatic wave equations in the Cowling approximation. A comparison between the results obtained by using the ray approximation and the spherical polar expansion is shown. We use the acoustic-gravity wave equation in the plane-parallel limit in both cases and for the later we take the asymptotic approximation for the radial part of the solution. We also show that the phase shift in the Green's functions due to the incorrect modeling of the uppermost layers of the Sun can be estimated from the eigenfrequencies of the normal oscillation modes.

Green's Functions

Traditional far-side seismic maps made use of Green's functions calculated using the (acoustic) ray theory. Formally this approximation is only valid for high frequencies and high angular degrees. The approach that we take here is to determine the Green's functions through their polar expansion. Although the method allows a full numerical computation both in the radial and angular variables, in the present work we have used the asymptotic approximation for the radial part. On the other hand the angular part of the solutions is (at least theoretically) better represented by this expansion.

In the Cowling and plane-parallel limits, the point-source wave equation for the scalar field $G_0 = -\rho^{-1/2} \delta p$ is given by (see Gough, 1993)

$$\nabla^2 G_0 - (N^2/\omega^2) \nabla^2_r G_0 + (1/c^2)(\omega^2 - \omega_p^2) G_0 = -\delta(\mathbf{r} - \mathbf{r}_0) \quad (1)$$

Taking the polar expansion:

$$G_0(\mathbf{r} | \mathbf{r}_0) = (1/4\pi) \sum g^l(\mathbf{r} | \mathbf{r}_0) (2l+1) P^l(\mu) \quad (2)$$

where \mathbf{r}_0 is the focus, $P^l(\mu)$ a Legendre polynomial and $\mu = \cos\theta \cos\theta_0 + \sin\theta \sin\theta_0 \cos(\varphi - \varphi_0)$, the general solution can be written as

$$r g^l(\mathbf{r}) = \alpha_+(r) A_+(r) + \alpha_-(r) A_-(r) \quad (3)$$

$$\sin^{1/2}\theta P^l(\theta) = \beta_+(\theta) B_+(\theta) + \beta_-(\theta) B_-(\theta) \quad (4)$$

- $A_+(\mathbf{r})$ and $A_-(\mathbf{r})$ are inward and outward asymptotic solutions of the radial wave equation while $B_+(\theta)$ and $B_-(\theta)$ are prograde and retrograde asymptotic solutions of the Legendre equation.

- The oscillating parameters α_+ , α_- , β_+ and β_- are smooth functions introduced to deal with non-asymptotic solutions while keeping the inward and outward decomposition.

- In computing the egression for the seismic image we only take the outward prograde part of the regular solution, $\alpha_+(r)A_+(r)\beta_+(\theta)B_+(\theta)$.

- To compute the Green's functions after m skips we introduce the corresponding phase shift at the focus.

The Phase Shift

Non adiabatic effects and the dynamics of convection are neglected in the wave equations. In the uppermost layers of the sun, these effects introduce a phase shift in the Green's functions of the form

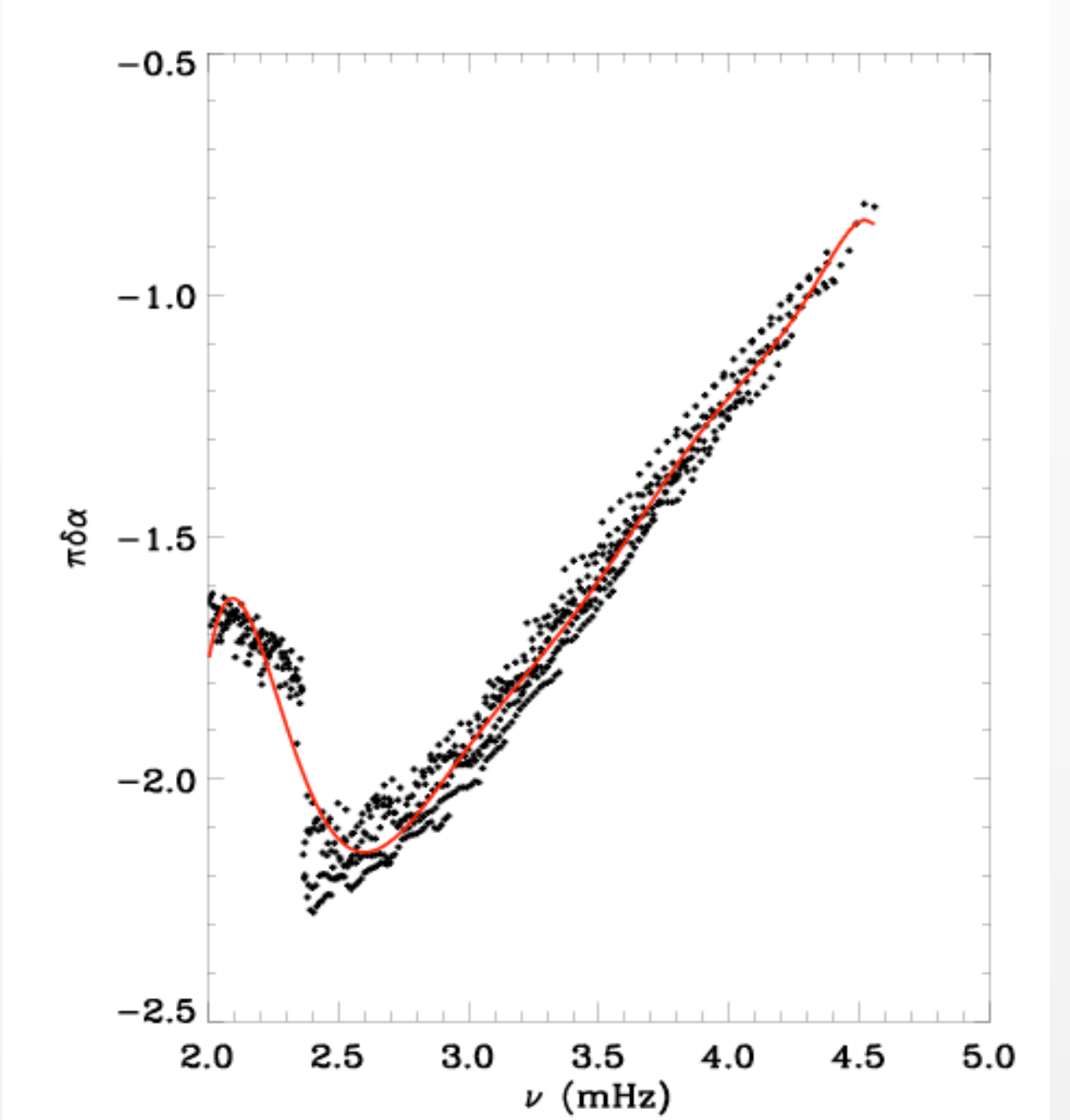
$$G(r, \theta) = G_0(r, \theta) \exp(2im\pi\delta\alpha) \quad (5)$$

where m is the number of skips at the surface and the phase shift $\delta\alpha$ is a function of frequency alone.

Provided observed eigenfrequencies are used, $\delta\alpha(\omega)$ can be estimated by

$$\int k_r dr = \pi(n - 1/2 - \delta\alpha) \quad (6)$$

where n is the radial order of the mode and k_r the radial part of the local wave vector. The figure on the right shows the results.

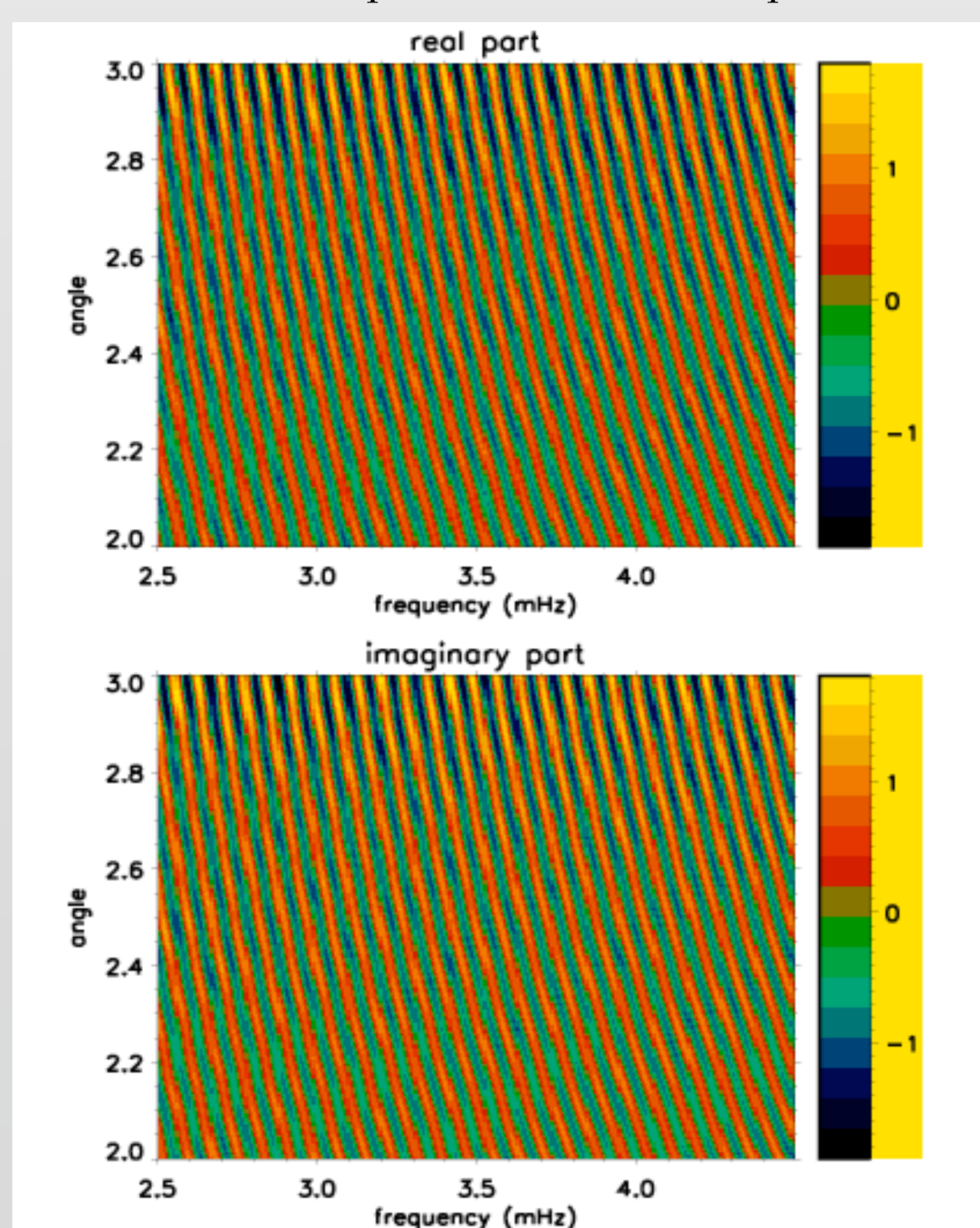


Phase shift computed using equation (6). Eigenfrequencies, with degrees $l=20-80$, are taken from MDI data. Model S of Christensen-Dalsgaard (1996) have been used. The solid thick red line represent a fitted polynomial of degree 10.

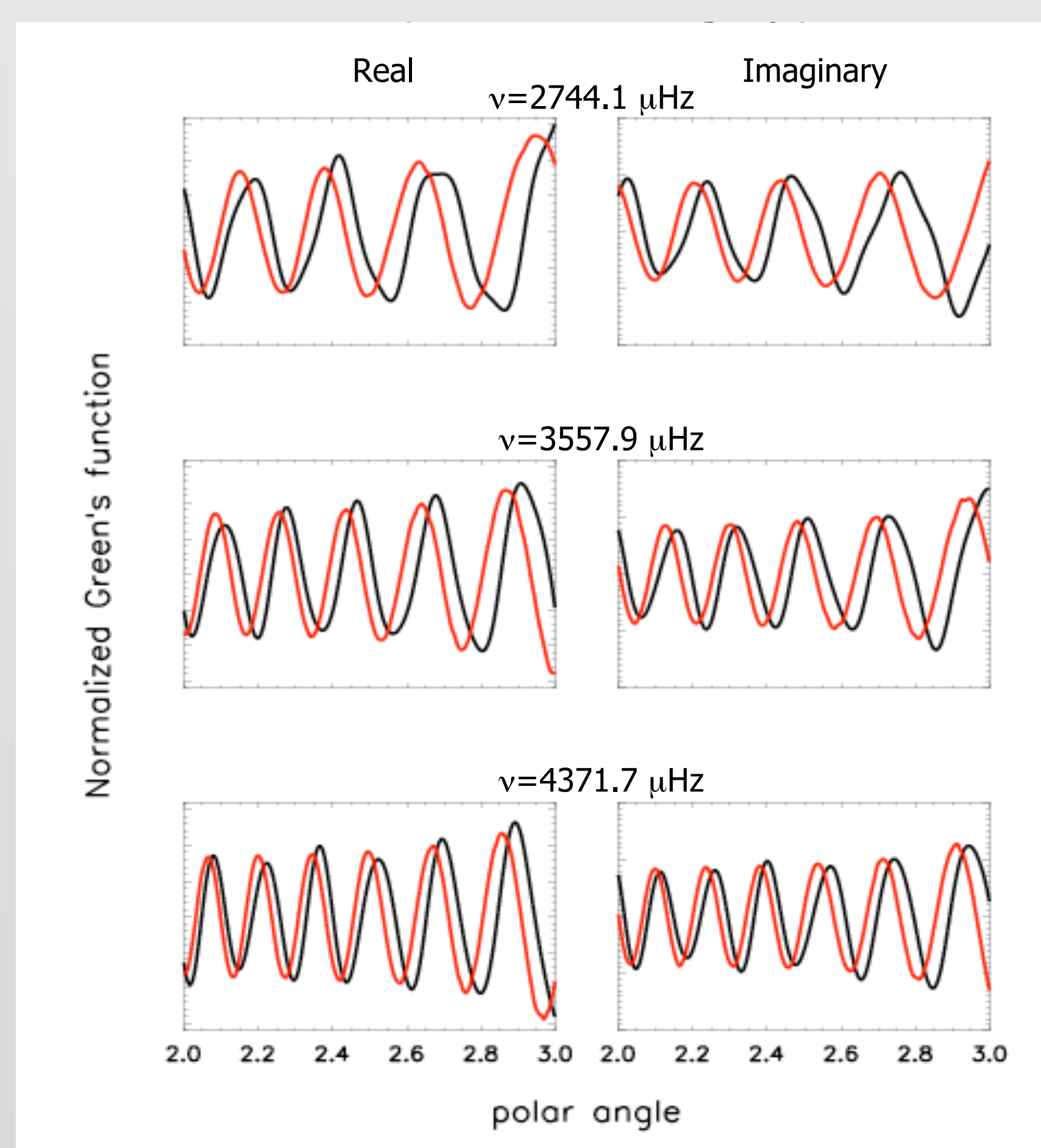
Ray theory versus polar expansion

The figure on the left bottom shows the ingoing Green's function as function of frequency and polar angle computed with a separation of variables as indicated above.

The figures on the right shows the Green's functions as function of the polar angle at the frequencies indicated. Black lines are for the polar expansion and red lines for the gravity-acoustic ray approximation. Notice that in both cases the wave equation is the same (Eq. 1 above).



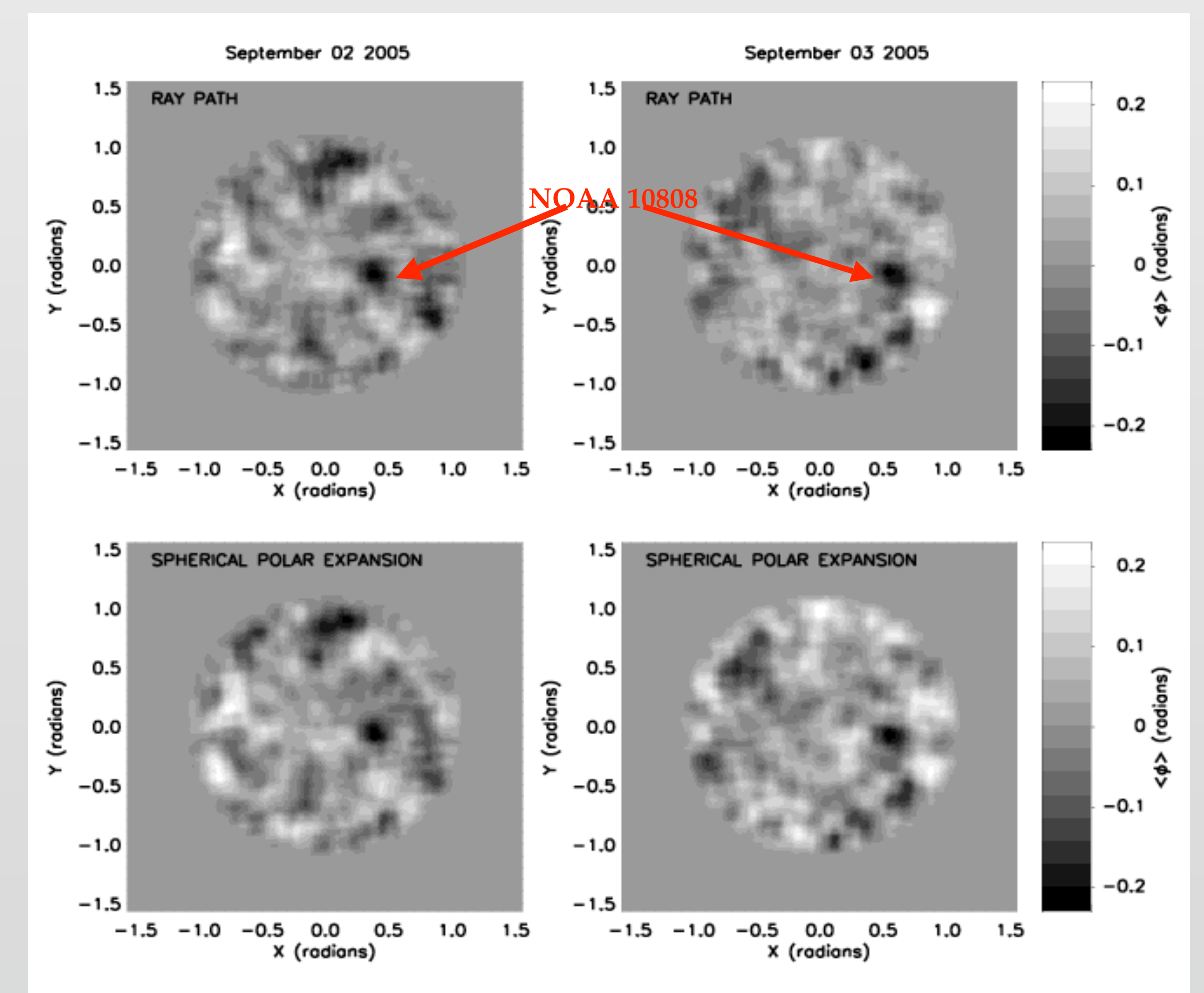
Green's functions: Real (top) and imaginary (bottom) part of Green's functions, as a function of frequency and skip distance from the focus, calculated using the Legendre polynomial decomposition. The focus is located at 0.9995 Solar Radius.



Comparison of Green's functions at selected frequencies. Black lines correspond to the Green's functions calculated using the Legendre polynomial decomposition and the red ones to those calculated using a ray path approximation.

Far-side maps

We use the new Green's functions to calculate maps of activity at the center of the non-visible hemisphere of the Sun from GONG Dopplergrams. The figures below present far-side maps for September 2 and 3 2005 clearly showing active region NOAA-10808. This active region appeared on the front side 4 days later.



Postel projected phase shift maps of the far side calculate using the acoustic-gravity ray approximation (top) and spherical polar expansion (bottom) Green's functions. The maps extend to approximately 50 degrees from the antipode of the center of the solar disk facing the Earth. The areas of large negative phase shift are related to areas of strong magnetic field.

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